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# Free Vibration of Multilayered Magneto-Electro-Elastic Plates With Skewed Edges Using Layer wise Shear Deformation Theory

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## Abstract

Present article discusses a novel method for the computation of non-dimensional eigen frequencies of a three dimensional multilayered magneto-electro-elastic plates (MEE) with skewed edges. A finite element (FE) model is formulated using a layerwise shear deformation theory (LSDT) and coupled constitutive equations. The transformation matrices are derived to transform local degrees of freedom into the global degrees of freedom for the nodes lying on the skew edges. Effect of different width to thickness ratios on the multilayered MEE plate with skewed edges is studied in detail. Particular attention has been paid to investigate the effect of various skew angles and stacking sequence on the non-dimensional eigen frequencies of multilayered MEE plate with simply supported boundary conditions.

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*Keywords:* Finite element; Layerwise shear deformation theory; Magneto-electro-elastic plates; Skew plate.

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## 1. Introduction

With growing demand for smarter composite structures in aerospace, marine application, sensors and actuators the studies related to multilayered smart composites involved in smart structures have captured much attention of the researchers recently. Such structural components with multilayered composites unveil several unique features. Hence, the structural response to the applied load has become an important research topic. Magneto-electro-elastic (MEE) composites are one amongst many smart structural composites composed of piezoelectric (BaTiO<sub>3</sub>) and magnetostrictive (CoFe<sub>2</sub>O<sub>4</sub>) materials. The MEE composites facilitate conversion of energy between electric and

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magnetic fields, termed as magneto-electric effect which is absent in individual constituents of the MEE composite (i.e. Pure piezoelectric and magnetostrictive phases). Though many researchers have contributed to the studies on MEE composites, Suchtelen [1] was first to report the magneto-electric effect in MEE composite. Later, the study of structural behavior with more highlight on plates, shells and beams were well reported. Pan and co-researchers [2-4] extensively investigated the static and vibrational behaviour of layered and functionally graded (FG) MEE composite structures. Exact deformation analysis of fiber reinforced MEE thin plates with closed circuit electric restriction was evaluated analytically by Liu [5]. Free vibrations of three dimensional multilayered MEE for clamped boundary condition was thoroughly investigated by Chen et al [6]. Kattimani and Ray [7-9] investigated on the geometrically nonlinear vibration control of MEE plates and shells using 1-3 piezoelectric composite. Ding and Jiang [10] investigated the simply supported annular MEE plate using the boundary element method. Ramirez et al. [11] adopted a discrete layer model to study the free vibration behaviour of MEE laminates and graded plates. Feng and Su [12] investigated the dynamics of FG MEE plate containing an internal crack. Simões [13] reported the static and free vibration of MEE plates using a higher - order model. Alaimo [14] discussed a novel FE formulation to study the large deflections in MEE multilayered plates. The semi-analytical solutions were developed by Xin and Hu [15] for the free vibration studies of multilayered MEE plates. Shooshtari and Razavi [16] studied the nonlinear free and forced vibration of a transversely isotropic rectangular MEE thin plate. Jun et al. [17] employed a semi-analytical model to investigate the deformations of MEE plate. Shooshtari and Razavi [18] investigated the free vibrations of MEE plate using Reddy's third-order shear deformation theory.

Plates and laminates with skewed edges find a prominent presence in many engineering applications. Composite plates with skewed edges exhibit high natural frequencies for the identical dimensions of the normal plate, thereby reducing the excess use of material. All these unique properties have been successful in attracting the attention of many researchers. Studies on free vibration of plates with skewed edges have been extensively carried out. Garg et al. [19] investigated vibration analysis of different skew laminates using a higher-order shear deformation theory. Kanasogi and Ray [20] have studied the active vibration control for different layups of the skew composite plate. The extensive literature review provides a larger insight over free vibration of MEE composite plates and laminated composite plates with skewed edges. However, multilayered MEE plates with skewed edges has not been investigated and provides an ample scope for further research. It is noteworthy to mention that to the best of the author's knowledge, the research concerning the three dimensional, multi-layered MEE plate with skewed edges has not been reported in the open literature. Hence, this paper presents the layerwise shear deformation theory (LSDT) to develop FE formulation for the free vibration studies of MEE plates with skewed edges. Effect of boundary condition, width to thickness ratio and stacking sequence on the free vibration behavior has been investigated thoroughly.

## 2. Problem description and governing equation

A schematic representation of magneto-electro-elastic plate with skewed edges having length  $a$ , width  $b$ , total thickness  $H$  and skew angle  $\alpha$  is depicted in Fig. 1. The multilayered MEE plate has the top and the bottom layers of similar material either  $\text{CoFe}_2\text{O}_4/\text{BaTiO}_3$ ; while, the middle layer is of either  $\text{BaTiO}_3/\text{CoFe}_2\text{O}_4$ . The Cartesian coordinate system is represented by  $(x, y, z)$  and  $(x_1, y_1, z_1)$  are the local coordinates at skew angle  $\alpha$  of the skewed MEE plate. The MEE plate with four stacking sequence is considered for the analysis. The layer stacking arrangement are pronounced as B/F/B, B/B/B, F/F/F and B/F/B/F/B in which B and F represent piezoelectric  $\text{BaTiO}_3$  (Barium titanate) and magnetostrictive  $\text{CoFe}_2\text{O}_4$  (Cobalt ferrite), respectively. Here, B/B/B and F/F/F configuration are the pure piezoelectric and the pure piezomagnetic plates, respectively while B/F/B and B/F/B/F/B configuration corresponds to the MEE plates.

### 2.2 Displacement field.

The substrate plate is multilayered with combination of dissimilar materials. Therefore, layerwise shear deformation theory (LSDT) has been implemented in the analysis. Based on the LSDT, the displacement fields can be expressed as

$$\begin{aligned} u_i(x, y, z) &= u_{0i}(x, y) + z \theta_i(x, y) \\ w(x, y, z) &= w_0(x, y) + z \theta_z(x, y) + z^2 \zeta_z(x, y) \end{aligned} \quad (1)$$

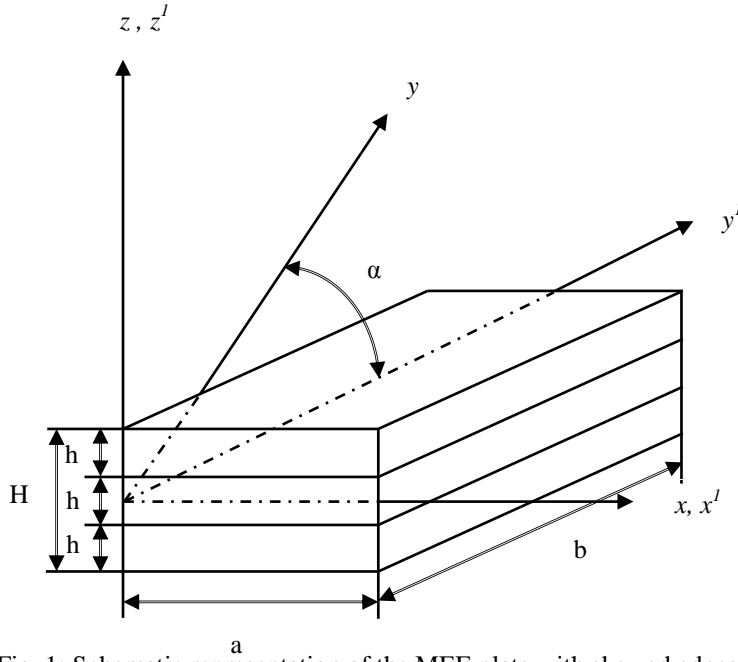


Fig. 1: Schematic representation of the MEE plate with skewed edges.

where  $i = x$  and  $y$ ,  $u_x = u$ ,  $u_y = v$ ,  $u_{0x} = u_0$ ,  $u_{0y} = v_0$ . In the above displacement fields,  $u_0$ ,  $v_0$  and  $w_0$  are the mid-plane displacements of the MEE plate with skewed edges while,  $\theta_x$  and  $\theta_y$  are shear displacements.  $\theta_z$  and  $\zeta_z$  refer to the rotational displacements about  $z$ - direction of the plate. As thin structures being more susceptible to shear locking, the strain component are considered separately for bending and shearing to study the effect of transverse shear deformation individually. The strain vectors associated with the displacement field in Eq. (1) at any point in the plate can be expressed as follows:

$$\{\epsilon_b^k\} = \{\epsilon_{bt}\} + [R_1]\{\epsilon_{rb}\}, \{\epsilon_s^k\} = \{\epsilon_{ts}\} + [R_2]\{\epsilon_{rs}\} \quad (2)$$

wherein  $k$  represents the layer number for the plate,  $[R1]$  and  $[R2]$  defines the transformation matrices; while the strain vectors appearing in Eq. (2) are given as follows:

$$\{\epsilon_{bt}\} = \begin{bmatrix} \frac{\partial u_0}{\partial x} & \frac{\partial v_0}{\partial y} & 0 & \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{bmatrix}, \{\epsilon_{ts}\} = \begin{bmatrix} \frac{\partial w_0}{\partial x} & \frac{\partial w_0}{\partial y} \end{bmatrix} \text{ and}$$

$$\{\epsilon_{rb}\} = \begin{bmatrix} \frac{\partial \theta_x}{\partial x} & \frac{\partial \theta_y}{\partial y} & \frac{\partial \theta_x}{\partial x} + \frac{\partial v_0}{\partial x} & \theta_z & \zeta_z \end{bmatrix}$$

### 2.3 Constitutive equation

The constitutive equations considering the effect of coupled fields, for the MEE plate with skewed edges can be written as

$$\{\sigma_b^k\} = [\bar{Q}_b^k]\{\epsilon_b^k\} - \{e_b^k\} E_z - \{q_b^k\} H_z, \{\sigma_s^k\} = [\bar{Q}_s^k]\{\epsilon_s^k\} \quad (3a)$$

$$D_z = \{e_b^k\}^T \{\epsilon_b^k\} + e_{33}^k E_z + d_{33}^k H_z \quad (3b)$$

$$B_z = \{q_b^k\}^T \{\epsilon_b^k\} + d_{33}^k E_z + \mu_{33}^k H_z \quad (3c)$$

where  $k = 1, 2, 3$  designates the layer number from the bottom layer to the top and

$$[\bar{Q}_b^k] = \begin{bmatrix} \bar{Q}_{11}^k & \bar{Q}_{12}^k & \bar{Q}_{13}^k & \bar{Q}_{16}^k \\ \bar{Q}_{12}^k & \bar{Q}_{22}^k & \bar{Q}_{23}^k & \bar{Q}_{26}^k \\ \bar{Q}_{13}^k & \bar{Q}_{23}^k & \bar{Q}_{33}^k & \bar{Q}_{36}^k \\ \bar{Q}_{16}^k & \bar{Q}_{26}^k & \bar{Q}_{36}^k & \bar{Q}_{66}^k \end{bmatrix}, [\bar{Q}_s^k] = \begin{bmatrix} \bar{Q}_{55}^k & \bar{Q}_{45}^k \\ \bar{Q}_{45}^k & \bar{Q}_{44}^k \end{bmatrix} \quad (4)$$

where,  $[\bar{Q}_b^k]$  and  $[\bar{Q}_s^k]$  being the transformed coefficient matrices,  $\epsilon_{33}^k$ ,  $\mu_{33}$  and  $d_{33}$  are the dielectric, the magnetic permeability and the electromagnetic coefficient, respectively.  $D_z$ ,  $E_z$ ,  $B_z$  and  $H_z$  represents the electric displacement, the electric field, the magnetic induction and the magnetic field respectively,  $\{e_b^k\}$  and  $\{q_b^k\}$  represents the electric coefficient matrix and the magnetic coefficient matrix, respectively.

## 2.4 Governing equation

Using the principle of virtual work, the governing equations for the MEE plate with skewed edges can be established as

$$\sum_{k=1}^3 \left( \int_{\Omega^k} \delta\{\epsilon_b^k\} \{\sigma_b^k\} d\Omega^k + \int_{\Omega^k} \delta\{\epsilon_s^k\} \{\sigma_s^k\} d\Omega^k + \int_{\Omega^k} \delta\{d_t\}^T \rho^k \{\ddot{d}_t\} d\Omega^k \right) - \int_{\Omega^t} \delta E_z^t D_z^t d\Omega^t - \int_{\Omega^b} \delta E_z^b D_z^b d\Omega^b - \int_{\Omega^m} \delta H_z B_z d\Omega^m = 0 \quad (5)$$

where,  $\Omega^k$  ( $k = 1, 2, 3$ ) indicates the volume of the respective layer,  $\rho^k$  denotes the mass density of the  $k^{\text{th}}$  layer. The superscript t, b and m in the above equation represents the variables corresponding to the top, bottom and the middle layers of the MEE plate with skewed edges. The transverse electric field and the electric potential, the transverse magnetic field and the magnetic potential related correspondingly in accordance with the Maxwell's equation. The interfaces between the piezoelectric and magnetostrictive layers are assumed to be suitably grounded. As the MEE plate considered is very thin, the variation of the electric potential and the magnetic potential functions are assumed to be linear across the thickness.

## 2.5 Skew boundary transformation

For the skewed MEE plates, the displacements along the skew edges lying in the local coordinate need to be transformed into the global coordinate to facilitate the proper imposition of boundary conditions. The transformation is achieved by the relations given as follows:

$$\{d_t\} = [L_t] \{d_t^l\}, \{d_r\} = [L_r] \{d_r^l\} \quad (6)$$

$$\{d_t^l\} = [u_0^1 \ v_0^1 \ w_0^1]^T, \{d_r^l\} = [\theta_x^1 \ \theta_y^1 \ \theta_z^1 \ \zeta_z^1]^T \quad (7)$$

War,  $d_r$  and,  $d_r^l$  are the generalized displacements on the global and the local edge coordinate system, respectively.

$[S_t]$  and  $[S_r]$  are the transformation matrices for a node on the skew boundary and are given by

$$[S_t] = \begin{bmatrix} m & n & 0 \\ -n & m & 0 \\ 0 & 0 & 1 \end{bmatrix}, [S_r] = \begin{bmatrix} m & n & 0 & 0 \\ -n & m & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

where,  $m = \cos\alpha$  and  $n = \sin\alpha$ . It may be noted that the transformation is not required for the nodes not lying on the skew edges and the diagonal elements of the transformation matrices are considered to be unity for such cases.

## 2.6 Free vibration behavior

Using equations (2) - (8) the final global equations of motion are obtained as follows:

$$\begin{aligned}
 [M]\{\ddot{d}_t\} + ([K_1] - [K_2][K_3]^{-1}[K_2]^T)\{d_t\} &= \{F_t\} \\
 [M]\{\ddot{d}_t\} + [K]\{d_t\} &= \{F_t\} \\
 \text{and } [K] &= ([K_1] - [K_2][K_3]^{-1}[K_2]^T) \tag{9}
 \end{aligned}$$

where, the global aggrandized matrices are given as follows:

$$\begin{aligned}
 [K_1] &= [k_{tt}^g] + [k_{t\phi}^g][k_{\phi\phi}^g]^{-1}[k_{t\phi}^g]^T + [k_{t\psi}^g][k_{\psi\psi}^g]^{-1}[k_{t\psi}^g]^T, \\
 [K_2] &= [k_{tr}^g] + [k_{t\phi}^g][k_{\phi\phi}^g]^{-1}[k_{r\phi}^g]^T + [k_{t\psi}^g][k_{\psi\psi}^g]^{-1}[k_{r\psi}^g]^T, \\
 [K_3] &= [k_{rr}^g] + [k_{r\phi}^g][k_{\phi\phi}^g]^{-1}[k_{r\phi}^g]^T + [k_{r\psi}^g][k_{\psi\psi}^g]^{-1}[k_{r\psi}^g]^T.
 \end{aligned}$$

The eigenvalue problem can be formulated considering the global equations of motion in terms of global translational degrees of freedom as follows:

$$[K] + \lambda[M] = 0 \tag{10}$$

where,  $[K]$  is the global stiffness matrix,  $[M]$  is the global mass matrix and  $\lambda$  is the eigenvalue i.e.,  $\lambda = \omega^2$ .

### 3. Results and discussions

The response of magneto-electro-elastic plate with skewed edges under simply supported boundary conditions are studied for free vibration behavior with various skew angles of the plate edges. The effect of stacking sequences and width to thickness ratio for different skew angles are presented. The mechanical properties of the constituent layers of the plate are considered from chen et al.[6]. The MEE square plate of width  $a$  and length  $b$  is discretized into  $4 \times 4$  mesh constituting 16 elements. Eight node iso-parametric quadrilateral serendipity elements devised the discretization with three translational DOF; two rotational DOF and an electric and magnetic potential DOF at each node. The layer thickness is taken as 0.1 m, whereas the length and the width are considered 1 m. The total plate thickness is considered to be same for all the stacking configuration. The non-dimensional natural frequencies are obtained using the relation

$$\bar{\omega} = \omega a \sqrt{\rho_{\max} / C_{\max}} \tag{11}$$

where,  $a$  = Length of the plate;  $\rho_{\max}$  = Maximum density;  $C_{\max}$  = Maximum elastic constant.

#### 3.1 Validation

The proposed FE model in the preceding section is verified with a semi-analytical model for the multilayered MEE plate [6]. The material properties are considered from Chen et al.[6]. The density of Barium Titanate (BaTiO3) and Cobalt Ferrite (CoFe2O4) are taken as 5800 kg/m<sup>3</sup>. The normalized eigen modes for the clamped-clamped boundary condition of the multi-layered MEE plate are extracted from the present FE model and compared with the reference results in Table 1. To the author’s best knowledge free vibration analysis of MEE skew plates are not available in open literature. Hence to verify the effectiveness of the developed model for the skew plates, non-dimensional natural frequencies of the skew laminated composites obtained by Garg et al. [20] is considered and compared. The corresponding natural frequencies for the skew composite plates are listed in Table 2. It may be observed from Table 1 and 2 that, the results obtained using the present finite element formulation are in very good agreement with results reported by Chen et al.[6] and Garg et al.[20].

#### 3.2 Eigen frequencies evaluation of skewed MEE plates

Having verified the present FE model with the multilayered MEE plate [6] and for the skew laminated composite plates [20], the present FE formulation is extended for the three layered MEE plate with skewed edges composed of the piezoelectric layer on the top and bottom while the magnetostrictive layer in the middle. The stacking sequences

considered for the analysis are B/F/B, B/B/B and F/F/F with B representing the piezoelectric material BaTiO<sub>3</sub> and F represents magnetostrictive CoFe<sub>2</sub>O<sub>4</sub>. The B/B/B and F/F/F stacking configuration are considered to emphasise the adaptability of the present FE model for the studies related to the pure piezoelectric and the pure piezomagnetic phases. The simply supported MEE plate with traction free surfaces on the top and the bottom is considered for the analysis. Table 3 and 4 enlist the non-dimensional eigen frequencies for B/F/B, B/B/B and F/F/F stacking sequences, respectively. It can be observed from the Table 3 that for the B/F/B stacking configuration eigen frequencies increase with the increase in skew angle of the MEE plate. Similar trend is observed for the B/B/B and F/F/F configurations in Table 4. It may also be observed from Table 3 and 4 that the eigen frequencies are higher for multi-phase and multilayered B/F/B stacking configuration over single phase B/B/B and F/F/F stacking configurations. Further, in order to study the effect of number of stacking layers, five layered B/F/B/F/B plate is also considered and the eigen frequencies are tabulated in Table 5. It may be noted that the number of layers has a significant effect on the eigen frequencies. It may also be noticed from Table 3 and 5 that the eigen frequencies of B/F/B plates are higher in comparison with that of B/F/B/F/B configuration for the skew angles above 60°.

Table 1: Non-dimensional frequency parameter  $\lambda = \omega b^2 / \pi^2 h (\rho/E_2)^{1/2}$  for the clamped-clamped laminated composite plate ( $a/h=10$ ).

Skew angle ( $\alpha$ )	Source	Antisymmetric cross-ply (0°/90°/0°/90°)			Symmetric cross-ply (90°/0°/90°/0°/90°)		
		Modes			Modes		
		1	2	3	1	2	3
0°	Ref. [20]	2.2990	3.7880	3.7880	2.3687	3.5399	4.1122
	Present	2.2590	3.5213	4.2695	2.2400	3.3655	4.2382
15°	Ref. [20]	2.3809	3.7516	4.0785	2.4663	3.6255	4.3418
	Present	2.2992	3.4560	4.2841	2.2860	3.3637	4.2346
30°	Ref. [20]	2.6666	3.9851	4.7227	2.7921	3.9557	5.0220
	Present	2.4403	3.5067	4.3609	2.4396	3.4363	4.2949
45°	Ref. [20]	3.3015	4.6290	5.8423	3.4739	4.7129	5.8789
	Present	2.7348	3.7102	4.6270	2.7439	3.6545	4.6862

Table 2: Non-dimensional normalized natural frequency modes for clamped-clamped B/F/B plate.

Non dimensional normalized natural frequency of B/F/B clamped-clamped plate $\bar{\omega} = \omega a \sqrt{\rho_{\max} / C_{\max}}$										
Source	1	2	3	4	5	6	7	8	9	10
Present	1.2685	2.0812	2.0812	2.6383	2.6383	2.7142	2.9750	3.0940	3.1286	3.5894
Ref. [6]	1.3452	2.2231	2.2231	2.6178	2.6178	2.9404	2.9939	3.3123	3.3758	3.7729

Table 3: Variation of non-dimensional frequencies ( $\bar{\omega} = \omega a \sqrt{\rho_{\max} / C_{\max}}$ ) for simply supported B/F/B MEE plate with skewed edges at different skew angles.

Skew angle ( $\alpha$ )	Non dimensional normalized natural frequencies for BFB under SSSS boundary condition with $a = b = 1$									
	1	2	3	4	5	6	7	8	9	10
0°	0.9412	1.2940	1.2940	1.8307	1.8728	1.8728	2.5515	2.5971	2.5971	2.9080
15°	0.9925	1.3322	1.3487	1.8493	1.8773	2.0515	2.5720	2.6542	2.6866	2.8718
30°	1.1679	1.4589	1.5420	1.9835	2.0316	2.4389	2.7093	2.8340	2.9846	2.9972
45°	1.5497	1.7251	1.9793	2.3489	2.3556	3.0974	3.1865	3.1966	3.3864	3.6112
60°	2.3060	2.3905	2.9802	3.0508	3.2060	3.9999	4.0945	4.3820	4.7572	4.8032
75°	4.1483	4.9280	5.0075	5.8770	5.9404	6.9208	7.5477	7.8193	8.2420	8.8379

### 3.3 Effect of width to thickness ratio on eigen frequencies

In this section, it is intended to study the effect of different width to thickness ratios on the non-dimensional eigen frequencies. The eigen frequencies for a/h ratios of 30, 100 and 200 corresponding to thin plates for the skew angle of  $\alpha = 60^\circ$  are tabulated in Table 6. From the tabulated results it may be observed that the eigen frequencies attain lower values for higher values of a/h ratios.

Table 4: Variation of non-dimensional frequencies ( $\bar{\omega} = \omega a \sqrt{\rho_{\max} / C_{\max}}$ ) for simply supported B/B/B MEE plate with skewed edges at different skew angles.

Skew angle ( $\alpha$ )	Non dimensional normalized natural frequencies for B/B/B and F/F/F under SSSS boundary condition with $a = b = 1$									
	B/B/B	F/F/F	B/B/B	F/F/F	B/B/B	F/F/F	B/B/B	F/F/F	B/B/B	F/F/F
	1	1	2	2	3	3	4	4	5	5
$0^\circ$	0.2772	0.3206	0.6196	0.6981	0.6196	0.6981	0.6313	0.7134	0.6313	0.7134
$15^\circ$	0.2977	0.3428	0.6235	0.7042	0.6377	0.7185	0.6435	0.7258	0.7091	0.7971
$30^\circ$	0.3684	0.4198	0.6872	0.7716	0.6976	0.7861	0.7257	0.8214	0.8846	0.9838
$45^\circ$	0.5278	0.5935	0.8224	0.9267	0.8593	0.9543	0.9054	1.0315	1.1059	1.2499
$60^\circ$	0.9088	1.0017	1.0892	1.2268	1.2724	1.3883	1.3019	1.4947	1.3933	1.5770
$75^\circ$	1.8969	2.1355	2.1444	2.2788	2.2066	2.5064	2.4356	2.7220	2.5673	2.7908

Table 5: Variation of non-dimensional frequencies ( $\bar{\omega} = \omega a \sqrt{\rho_{\max} / C_{\max}}$ ) for simply supported B/F/B/F/B MEE plate with skewed edges at different skew angles.

Skew angle ( $\alpha$ )	Non dimensional normalized natural frequencies for B/F/B/F/B under SSSS boundary condition with $a = b = 1$									
	1	2	3	4	5	6	7	8	9	10
$0^\circ$	1.1999	1.2723	1.2723	1.7999	2.1204	2.1204	2.5530	2.5530	2.7430	2.8585
$15^\circ$	1.2500	1.3096	1.3257	1.8453	2.0960	2.2852	2.6060	2.6399	2.7596	2.8219
$30^\circ$	1.4185	1.4334	1.5147	1.9958	2.2153	2.6358	2.7760	2.8784	2.9298	2.9427
$45^\circ$	1.6935	1.7765	1.9425	2.3122	2.5433	3.1260	3.2198	3.3033	3.3215	3.5413
$60^\circ$	2.2611	2.5506	2.9227	2.9906	3.3162	3.9191	4.1083	4.2953	4.6966	4.7155
$75^\circ$	4.0615	4.9043	4.9064	5.7723	5.8085	6.7918	7.2459	7.6908	7.8359	8.6831

Table 6: Variation of non-dimensional frequencies ( $\bar{\omega} = \omega a \sqrt{\rho_{\max} / C_{\max}}$ ) for simply supported B/F/B MEE plate with skewed edges at different width to thickness ratios at  $\alpha = 60^\circ$

a/h ratio	Non dimensional normalized natural frequencies for BFB under SSSS boundary condition with $\alpha = 60^\circ$									
	1	2	3	4	5	6	7	8	9	10
<b>30</b>	0.6659	1.0061	1.5722	1.8275	2.3033	2.3053	2.5386	2.9598	3.0396	3.5332
<b>100</b>	0.2286	0.3657	0.6243	0.6491	0.9469	1.1303	1.2758	1.4624	1.7189	1.7863
<b>200</b>	0.1211	0.2126	0.3259	0.3958	0.5424	0.6676	0.6717	0.7944	1.0092	1.0966

### Conclusion

In the present paper, a FE model is developed to study the free vibration behaviour of MEE plate with skewed edges using layerwise shear deformation theory. Effect of various parameters such as skew angle, stacking sequence and aspect ratio on the non-dimensional eigen frequencies were investigated. For all stacking configurations considered, the eigen frequencies were found to increase significantly for higher skew angles which is attributed to the increased stiffness of the MEE plate. The eigen frequencies are higher for  $\alpha \leq 45^\circ$  in case of B/F/B/F/B stacking sequence while for B/F/B stacking sequence eigen frequencies are higher for  $\alpha \geq 60^\circ$ . Further, a decreasing trend in eigen frequencies with higher width to thickness ratio is observed. It may be due to the fact that thinner plates possess lower stiffness.

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