FINITE ELEMENT FORMULATION FOR PASSIVE SHAPE CONTROL OF THIN COMPOSITE PLATES WITH INTEGRATED PIEZOELECTRIC LAYER

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Abstract. The Hamilton's principle for piezoelectric materials and the strain displacement relations based on the classical laminate theory's kinematics of deformation are utilized in deriving the piezoelectroelastic finite element equations of motion. Lagrange interpolation functions for in-plane displacement and Hermite cubic shape functions (conforming type) for transverse deflection are implemented through a four noded rectangular element. The formulation does not account voltage as the nodal degree of freedom. The computer code developed for composite plates with integrated piezoelectric sensors and actuator layers has been extensively validated for piezoelectric behaviour, static deflection and free vibration. The laminate deflection suppressed depends on the magnitude of the voltage applied, and this is a passive method of shape control. The effect of fibre orientation, stacking sequence and number of plies has been part of the numerical exercise on passive shape control.

Keywords: electric enthalpy, fibre-reinforced composite plate, Lagrange and Hermite cubic shape functions, actuating voltage, passive shape control

1. Introduction

A smart structure or "intelligent" structural system possesses self-sensing, diagnosis and control capabilities. Literature on the finite element formulation for the analysis of application of piezoelectric material for vibration, buckling and

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deflection suppression is quite vast. Variational principle which incorporates piezoelectric effect has been used to develop tetrahedral finite element for the electroelastic vibration analysis of piezoelectric continuum[1]. The shape control and active vibration suppression of laminated composite plate with piezoceramic layers has been analyzed using non-conforming laminate finite element[2]. Based on the element free Galerkin method a mesh free finite element based on first order shear deformation theory for the static analysis of laminated composite beams and plates with integrated piezoelectric layers has been the focus of study in [3]. A new piezoelectric plate bending element with electric degree of freedom was developed for analysis of static shape control[4].

2. Theoretical Formulation: Finite element formulation for thin laminates with piezolayers and finite element equations for passive shape control

The dynamic equations of a piezoelectric continuum can be derived from the Hamilton's principle in conjunction with the linear piezoelectric constitutive equations[1,5]. The Lagrangian L of the Hamiltonian is the sum of kinetic energy J and electrical enthalpy H (the sum of the strain and electric energies for piezoelectric materials) and also includes the virtual work of mechanical and electrical forces. Using the standard finite element procedure: The shape functions $\mathbf{N}_\mathbf{u}$ relating the displacement field, $\mathbf{u}$, to the nodal displacement values $\bar{\mathbf{u}}$ and the shape function $\mathbf{N}_\phi$ relating the electric field to the nodal electric potential are used to derive the finite element equation of motion for piezoelectric continuum. The strain field $\boldsymbol{\varepsilon}$ are expressed in terms of the derivatives of the nodal displacement vector $\bar{\mathbf{u}}$ by using the differential operator matrix, and the electric field vector $\mathbf{E}$ is defined by the electric potential $\bar{\phi}$ by the gradient operator. The corresponding statement for a finite element is:

$$
\begin{aligned}
0 = & -\delta\dot{\bar{\mathbf{u}}}^T \int_V \rho \mathbf{N}_u^T \mathbf{N}_u \, dV \ddot{\bar{\mathbf{u}}} - \delta\bar{\mathbf{u}}^T \int_V \mathbf{B}_u^T \mathbf{c}^E \mathbf{B}_u dV \bar{\mathbf{u}} - \delta\bar{\mathbf{u}}^T \int_V \mathbf{B}_u^T \mathbf{e} \mathbf{B}_\phi dV \bar{\phi} - \delta\bar{\phi}^T \int_V \mathbf{B}_\phi^T \mathbf{e}^T \mathbf{B}_u dV \bar{\mathbf{u}} \\
& + \delta\bar{\phi}^T \int_V \mathbf{B}_\phi^T \boldsymbol{\chi} \mathbf{B}_\phi dV \{\phi_i\} + \delta\bar{\mathbf{u}}^T \int_V \mathbf{N}_u^T \mathbf{F}_b dV + \delta\bar{\mathbf{u}}^T \int_{S_1} \mathbf{N}_u^T \mathbf{F}_S dS + \delta\bar{\mathbf{u}}^T \mathbf{N}_u^T \mathbf{F}_P \\
& - \delta\bar{\phi}^T \int_{S_2} \mathbf{N}_\phi^T \varsigma dS - \delta\bar{\phi}^T \mathbf{N}_\phi^T Q
\end{aligned} \tag{1}
$$

$\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain vectors, $\mathbf{c}$ is the elastic constant matrix, $\mathbf{e}$ is the piezoelectric stress coefficient matrix ($\mathbf{e}^T$ is the transpose of $\mathbf{e}$), $\boldsymbol{\chi}$ is the