Deploy and Search Strategy for Multi-agent systems using Voronoi partitions

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Abstract

In this paper we analyze a deploy and search *strategy for multi-agent systems. Mobile agents equipped with sensors carry out search operation in the search space. The lack of information about the search space is modeled as an uncertainty density distribution over the space, and is assumed to be known to the agents* a priori. *In each step, the agents deploy themselves in an optimal way so as to maximize per step reduction in the uncertainty density. We analyze the proposed strategy for convergence and spatial distributedness. The control law moving the agents has been analyzed for stability and convergence using LaSalle's invariance principle, and for spatial distributedness under a few realistic constraints on the control input such as constant speed, limit on maximum speed, and also sensor range limits. The simulation experiments show that the strategy successfully reduces the average uncertainty density below the required level.*

1 introduction

The problem of searching for targets in unknown environments has been addressed in the literature in the past [1]-[4]. These fundamental works were mostly theoretical in nature and were applicable to a single agent searching for single or multiple, static or moving, targets. It is likely that the same task can be accomplished more effectively by multiple searchers. But when multiple agents are involved, coordination between them becomes an important issue. Most biological systems such as ants, birds, fishes etc., have distributed local decision making capabilities which, in turn, lead to a useful collective behavior such as swarms, schools, flocks, etc. With each agent taking decisions based on only available local information and distributed control law, usually referred to as 'behavior' in biological systems, can lead to coordination among the agents and result in a meaningful collective behavior. These ideas of distributed control have been used widely in multi-agent systems. The distributed multi-agent systems have been shown to achieve and maintain the geometric formations, move as flocks while avoiding obstacles, thus mimicking their biological counterparts.

In multi-agent systems it is important to come up with distributed control laws that guarantee the stability and convergence to the desired collective behavior, under limited information and evolving network configurations. We can find in the literature, attempts to provide formal analytical results for proposed distributed control laws by some authors (see [6], [9]-[13] and the references therein).

Sujit and Ghose [7] propose the partitioning of the search space into hexagonal cells and, associate each cell with an uncertainty value representing the lack of information about the cell. As the agents move through these cells, they acquire information, reducing the corresponding uncertainty value. Cortes et al. [9, 10] use the concept of centroidal Voronoi configuration for optimal deployment of multiple agents in a convex hull with known uncertainty density function. Some of the concepts that were used by these authors commonly appear in locational optimization, quantization theory, and geometric optimization (see references in [9, 10]).

In [14] we used the idea of partitioning the search space from [7]. A variation of the centroidal Voronoi configuration used in [9, 10] was used in [14] to present some preliminary results on a multi agent search strategy. In this paper, we call the strategy as *deploy and search* strategy and present the complete analysis of the control law for stability, convergence, and spatial distributedness under some realistic constraints such as constant speed, limit on maximum speed, and sensor range limits. The strategy itself is

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also analyzed for convergence and spatial distributedness. We present a few simulation experiments with various parameters.

1.1 Organization of the paper

In Section 2 we provide the problem formulation. We discuss the *deploy and search* strategy in Section 3. The multi-center objective function for optimal deployment, selection of the sensor detection function, the critical points of the objective function, proportional control law for deploying the agents optimally are discussed in detail in this section and we also provide corresponding analytical results related to spatial distributedness, stability and convergence. Effect of a few realistic constraints such as agents moving with constant speed, maximum speed limit of agents, and sensor range limitation on the search strategy are discussed in Section 4, along with analysis of convergence and spatial distributedness of corresponding control laws. We discuss a few implementation issues in the Section 5. The simulation results and discussions are provided in Section 6, and finally Section 7 concludes the paper.

2 Multi-agent search

In this section we present the problem addressed in this paper. The problem is similar to that described in [14]. N agents perform search operation in an unknown environment. The lack of information is modeled as an uncertainty density distribution over the search space Q. The problem addressed in this paper is of deploying the N agents in Q to collect information, thereby reducing the uncertainty density distribution over Q. The problem formulation is stated formally as

1. $Q \subset \mathbb{R}^n$ is a compact, convex polytope and is the search space

2. $\phi : Q \mapsto [0,1]$ defines the density function representing uncertainty (lack of information). For a point $q \in Q$, $\phi(q) = 1(0)$ implies no (complete) information is available about $q \in Q$. By information we mean, knowledge about existence of the target that is being searched for.

3. N agents, equipped with sensors and communication equipments, deploy themselves in Q, and gather information, thus reducing the uncertainty.

4. $P(t) = \{p_1(t), p_2(t), \ldots, p_N(t)\} \subset Q$ denotes the configuration of the multi-agent system at time t, and $p_i(t)$ denotes the position of i-th agent at time t. In future, for convenience, we drop t and refer to the positions by just p_i.

5. Sensor's effectiveness reduces with Euclidean distance.

6. Ideally, we are looking for an optimal way of utilizing the agents to acquire complete information about Q, and thus have $\phi(q) = 0, \forall q \in Q$.

The search task is eventually gathering information (like looking for the targets) about the search space Q, leading to reduction in uncertainty density. Formally, the search task is defined by updating the uncertainty density as,

$$\phi_{n+1}(q) = \phi_n(q) \min_i \{\beta(\|p_i - q\|)\} \tag{1}$$

where, $\phi_n(q)$ is the density function at the n-th iteration, $\beta : \mathbb{R}_0 \mapsto [0,1]$, a function of Euclidian distance of a given point in space from the agent ($\mathbb{R}_0$ is the set of non-negative real numbers), acts as the factor of reduction in uncertainty by the sensors, and p_i is the position of the i-th sensor. At a given $q \in Q$, only the agent with the smallest $\beta(\|p_i - q\|)$, that is, agent which can reduce the uncertainty by the largest amount is active. It is clear that $\beta \in [0,1]$.

3 Deploy and search strategy

In this section we discuss the strategy proposed in [14] and formally name it as *deploy and search* strategy. The agents are deployed in Q in an optimal way to perform the search operation to acquire knowledge about the search space. This optimal deployment of agents is discussed in the following sections. The entire iteration of *deploy and search* continues till the density distribution is below the acceptable limit (such as maximum value of average density or a limit on $\max_{q \in Q}\{\phi(q)\}$, as specified by the problem).

3.1 Objective function (One-Step)

We are looking for deployment of agents in Q, maximizing per iteration reduction in the uncertainty ϕ. Consider the following objective function to be maximized.

$$\begin{aligned} \mathcal{H}_n &= \int_Q \Delta\phi_n(q) dQ \\ &= \int_Q \max_i\{(|\phi_n(q) - \beta(\|p_i - q\|)\phi_n(q)|)\} dQ \\ &= \int_Q (\phi_n(q) - \min_i\{\beta(\|p_i - q\|)\}\phi_n(q)) dQ \\ &= \sum_i \int_{V_i} \phi_n(q)(1 - \beta(\|p_i - q\|)) dQ \end{aligned} \tag{2}$$

where, V_i is the Voronoi partition corresponding to the i-th agent, and $p_i \in Q$ is the position of the i-th agent.

The gradient is given by [9]

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$$\frac{\partial \mathcal{H}_n}{\partial p_i} = \int_{V_i} \phi_n(q) \frac{\partial}{\partial p_i}[1 - \beta(r)] dQ \tag{3}$$

where, $r = \|p_i - q\|$

Theorem 1 *The gradient given by (3) is spatially distributed over the Delaunay graph $\mathcal{G}_D$.*

Proof. The gradient (3) defined for $p_i \in P$, the present configuration depends only on corresponding Voronoi partition V_i and values of ϕ and β within V_i. The Voronoi partition V_i depends only on the neighbors $\mathcal{N}_{\mathcal{G}_D}(p_i, P)$ of p_i. Thus, the gradient (3) can be computed with only local information, that is, the neighbors of p_i in $\mathcal{G}_D$. □

3.2 Selection of β

Here, $\beta : \mathbb{R} \mapsto [0, 1]$ is a non-decreasing function capturing effectiveness of the sensor. A continuously differentiable function leads to a possible closed-form solution to the optimization problem (2). Consider

$$\beta(r) = 1 - ke^{-\alpha r^2}, \quad k \in (0, 1] \quad \text{and } \alpha > 0$$

Here, $ke^{-\alpha r^2}$ represents the sensitivity of the sensor which is maximum at $r = 0$ and tends to zero as $r \to \infty$ and, β is minimum at $r = 0$ (effecting maximum reduction in ϕ) and tends to unity as $r \to \infty$ (change in ϕ reduces to zero as r increases). The parameter k gives the maximum sensitivity or effectiveness of the sensor and α specifies the rate at which the sensor effectiveness decreases with range. Higher the α, lower is the sensor effectiveness for a given range, that is, the sensitivity decreases faster with the distance.

3.3 Optimal solution

The objective function (2) will now take the form

$$\mathcal{H}_n = \sum_i \int_{V_i} \phi_n(q) ke^{-\alpha r^2} dQ \tag{4}$$

The gradient with respect to p_i is,

$$\begin{aligned} \frac{\partial \mathcal{H}_n}{\partial p_i} &= \textstyle\sum_i \int_{V_i} \phi_n(q) ke^{-\alpha(\|p_i - q\|)^2} (-2\alpha)(p_i - q) dQ \\ &= -2\alpha \tilde{M}_{V_i}(p_i - \tilde{C}_{V_i}) \end{aligned} \tag{5}$$

where $\tilde{M}_{V_i}$ and $\tilde{C}_{V_i}$ are the mass and the centroid of V_i, respectively, with respect to $\tilde{\phi}_n(q) = \phi_n(q) ke^{-\alpha r^2}$, which is the density as perceived by the sensor.

Thus the necessary condition for optimality is,

$$p_i = \tilde{C}_{V_i} \tag{6}$$

Note that $\tilde{C}_{V_i}$ depends on p_i, as the Voronoi partitions themselves depend on the agent configuration P. Thus, as the agents move, the Voronoi partitions are recomputed. Hence, we are interested in a fixed point of $\tilde{C}_{V_i}(p_i)$. When all the agents are located at the centroids of the corresponding Voronoi partitions with $\tilde{\phi}$ as the density, we call such an agent configuration as the *centroidal Voronoi configuration*.

3.4 The control law

Let us consider the system dynamics as

$$\dot{p}_i = u_i \tag{7}$$

Consider the control law

$$u_i = -k_{prop}(p_i - \tilde{C}_{V_i}) \tag{8}$$

Control law (8) moves the agent towards $\tilde{C}_{V_i}$ for positive k_{prop}, the proportional gain.

Theorem 2 *The trajectories of the agents governed by the control law (8), starting from any initial condition $P(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}_n$.*

Proof. Consider $V(P) = -\mathcal{H}_n$, where $P = (p_1, p_2, ..., p_N)$ represents the configuration of N agents.

$$\begin{aligned} \dot{V}(P) &= -\tfrac{d\mathcal{H}_n}{dt} \\ &= -\textstyle\sum_i \frac{\partial \mathcal{H}_n}{\partial p_i} \dot{p}_i \\ &= 2\alpha \textstyle\sum_i \tilde{M}_{V_I}(p_i - \tilde{C}_{V_i})(-k_{prop}(p_i - C'_{V_i})) \\ &= -2\alpha k_{prop} \textstyle\sum_i \tilde{M}_{V_i}(p_i - \tilde{C}_{V_i})^2 \end{aligned} \tag{9}$$

We observe that

1. $V : Q \mapsto \mathbb{R}$ is continuously differentiable in Q.
2. $M = Q$ is a compact invariant set.
3. $\dot{V}$ is negative definite in M.
4. $E = \dot{V}^{-1}(0) = \{\tilde{C}_{V_i}\}$.
5. E itself is the largest invariant subset of E by the control law (8).

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Here we will use LaSalle's invariance principle [15], which is basically an extension of Lyapunov's theorem [16, 17] requiring $\dot{V}$ to be *negative semi definite* rather than *negative definite* as in Lyapunov's theorem, and the candidate function V need not be *positive definite*.

Thus, by LaSalle's invariance principle [16, 17] the trajectories of the agents governed by control law (8), starting from any initial configuration $P(0) \in Q^N$, will asymptotically converge to set N, the critical points of $\mathcal{H}_n$, that is, the centroidal Voronoi partitions with respect to the density function as perceived by the sensors. □

Theorem 3 *The control law (8) is spatially distributed over the Delaunay graph $\mathcal{G}_D$.*

Proof. The control law uses $\tilde{C}_{V_i}$ which, in turn, depends only on the information available within V_i. As already discussed earlier in the proof of Theorem 1, V_i depends only on the neighbors $\mathcal{N}_\mathcal{G}(p_i, P)$ of agent i. □

Theorem 4 *The* deploy and search *strategy is spatially distributed over the Delaunay graph $\mathcal{G}_D$.*

Proof. The control law moving agents towards the centroids is spatially distributed by Theorem 3. The uncertainty density update function computation is done again within the corresponding Voronoi partition, and hence is spatially distributed over $\mathcal{G}_D$. □

Theorem 5 *The* deploy and search *strategy can reduce the average uncertainty to any arbitrarily small value.*

Proof. Look at the uncertainty density update law (1) for any $q \in Q$,

$$\begin{aligned} \phi_n(q) &= (1 - ke^{-\alpha r_i{}^2})\phi_{n-1}(q) \\ &= \gamma_{n-1}\phi_{n-1}(q) \end{aligned} \tag{10}$$

where, r_i is the distance of point $q \in Q$ from the i-th agent, such that $q \in V_i$, the Voronoi partition corresponding to it and, $\gamma_{n-1} := (1 - ke^{-\alpha r_i{}^2})$

Applying the above update rule recursively, we have,

$$\phi_n(q) = \gamma_{n-1}\gamma_{n-2}\ldots\gamma_1\gamma_0\phi_0(q) \tag{11}$$

Let $D(Q) := max_{p,q\in Q}(\parallel p - q \parallel)$. It should be noted that

(i) $0 \leq k < 1$

(ii) $0 \leq r_i \leq D(Q)$. $D(Q)$ is bounded as the set Q is bounded.

(iii) $0 \leq \gamma_j \leq 1 - ke^{-\alpha\{D(Q)^2\}} = l$ (say), $j \in \mathbb{N}$; and $l < 1$

Now consider the sequence $\Gamma = \{\Gamma_0, \Gamma_!, \Gamma_2, \ldots\}$, with

$$\Gamma_n := \gamma_n\gamma_{n-1}\ldots\gamma_1\gamma_0 \leq l^{n+1}$$

Taking limits as $n \to \infty$,

$$\lim_{n\to\infty} \Gamma_n \leq \lim_{n\to\infty} l^{n+1} = 0$$

Thus,

$$\lim_{n\to\infty} \phi_n(q) = \lim_{n\to\infty} \Gamma_{n-1}\phi_0(q) = 0$$

As the uncertainty density ϕ vanishes at each point $q \in Q$ in the limit, the average uncertainty density over Q is bound to vanish in the limit as $n \to \infty$. □

Remark: Note that the above proof does not depend on the control law. The theorem depends only on the outcome of the choice of the updating function (1) and the fact that there is no sensor range limitation and that the search space Q is bounded. In addition, the theorem does not address the issue of optimality of the strategy which, in fact, depends on the control law which is responsible for the movement of the agents.

4 Realistic constraints

In this section we analyze the proposed strategy in the presence of a few realistic constraints.

4.1 Maximum speed constraint

Consider a control law that takes into account the constraint on maximum speed of the agents denoted by U_{maxi}, for $i = 1, \ldots, n$, consider the control law

$$u_i = \begin{cases} -k_{prop}(p_i - \tilde{C}_{V_i}) & \text{If } u_i \leq U_{maxi} \\ -U_{maxi}\frac{(p_i - \tilde{C}_{V_i})}{\|(p_i - \tilde{C}_{V_i})\|} & \text{Otherwise} \end{cases} \tag{12}$$

The control law (12) makes the agents move towards their respective centroids with saturation on speed.

Theorem 6 *The trajectories of the agents governed by the control law (12), starting from any initial condition $P(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}_n$.*

Proof. Consider $V(P) = -\mathcal{H}_n$, where $P = (p_1, p_2, ..., p_N)$ represents the configuration of N agents.

$$\dot{V}(P) = \begin{cases} -2\alpha k_{prop} \sum_i \tilde{M}_{V_i}(p_i - \tilde{C}_{V_i})^2 & \text{, If } u_i \leq U_{maxi} \\ -2\alpha U_{maxi} \sum_i \tilde{M}_{V_i} \frac{(p_i - \tilde{C}_{V_i})^2}{\|(p_i - \tilde{C}_{V_i})\|} & \text{, otherwise} \end{cases} \quad (13)$$

We observe that the conditions similar to Theorem 2 are valid here, with E itself being the largest invariant subset of E by the control law (12).

Thus, as before, by LaSalle's invariance principle [16, 17], the trajectories of the agents governed by control law (12), starting from any initial configuration $P(0) \in Q^N$, will asymptotically converge to the set N, the critical points of $\mathcal{H}_n$, that is, the centroidal Voronoi partitions with respect to the density function as perceived by the sensors. □

4.2 Constant speed control

If the agents move with a constant speed U_i, for $i = 1, \ldots, n$, then we have the control law

$$u_i = \begin{cases} -U_i \frac{(p_i - \tilde{C}_{V_i})}{\|(p_i - \tilde{C}_{V_i})\|}, \text{ if } p_i \neq C_{V_i} \\ 0, \text{ otherwise} \end{cases} \quad (14)$$

The control law (14) moves the agents towards their respective centroids with a constant speed of $U_i > 0$.

Theorem 7 *The trajectories of the agents governed by the control law (14), starting from any initial condition $P(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}_n$.*

Proof. Consider $V(P) = -\mathcal{H}_n$, where $P = (p_1, p_2, ..., p_N)$ represents the configuration of N agents.

$$\dot{V}(P) = \begin{cases} -2\alpha U_i \sum_i \tilde{M}_{V_i} \frac{(p_i - \tilde{C}_{V_i})^2}{\|(p_i - \tilde{C}_{V_i})\|}, \text{ if } p_i \neq C_{V_i} \\ 0, \text{ otherwise} \end{cases} \quad (15)$$

We observe that the conditions similar to Theorem 2 are valid here, with E itself being the largest invariant subset of E by the control law (14).

Thus, again by LaSalle's invariance principle [16, 17], the trajectories of the agents governed by control law (14), starting from any initial configuration $P(0) \in Q^N$, will asymptotically converge to the set N, the critical points of $\mathcal{H}_n$, that is, the centroidal Voronoi partitions with respect to the density function as perceived by the sensors. □

4.3 Effect of sensor range limitation

In reality it is unlikely that the sensors will have infinete range. The sensors, in addition to having a monotonically decreasing sensitivity with Euclidean distance, might be totally insensitive to signals at distances larger than R, based on the sensor range. It can also be thought of as, when the sensitivity falls below say 5% of that of the maximum value, for all practical purposes, it can be assumed to be ineffective. There are two ways of modeling this phenomenon mathematically. A realistic one is $\forall r \geq R$, $f(r) = 0$, that is, $\forall r \geq R$, $\beta(r) = 1$. The agents can not reduce the density at points which are farther than R from them. An approximate one, which is much easier to handle, in mathematical analysis is $\forall r \geq R$, $f(r) = f(R)$, or $\beta(r) = \beta(R)$, which means that the sensor effectiveness gets saturated at $f(R)$. This is acceptable as long as $f(R)/f(0) \simeq 0$.

We shall look at the objective function with a saturation on β. Let

$$\hat{\beta}(r) = \begin{cases} \beta(r), \text{ if } r < R \\ \beta(R), \text{ otherwise} \end{cases} \quad (16)$$

Consider an objective function $\hat{\mathcal{H}}$ defined by

$$\hat{\mathcal{H}} = \sum_i \int_{(V_i \cap \bar{B}(p_i, R))} \phi_n(q)(1 - \hat{\beta}(\|p_i - q\|))dQ \quad (17)$$

It is easy to show (see Remark 2.3 in [10]) that the gradient of the objective function with the new updating function $\hat{\beta}$ is

$$\frac{\partial(\hat{\mathcal{H}})}{\partial p_i}(P) = 2\tilde{M}_{(V_i \cap \bar{B}(p_i, R))}(\tilde{C}_{(V_i \cap \bar{B}(p_i, R))} - p_i) \quad (18)$$

where, the mass $\tilde{M}$ and the centroid $\tilde{C}$ are now computed within the region $(V_i \cap \bar{B}(p_i, R))$, that is, the region of Voronoi partition V_i, which is accessible to the i-th agent. The critical points are nothing but $p_i = \tilde{C}_{(V_i \cap \bar{B}(p_i, R)}$.

The control law moving agents towards the new critical point is

$$u_i = -k_{prop}(p_i - \tilde{C}_{(V_i \cap \bar{B}(p_i, R))}) \quad (19)$$

Theorem 8 *The control law (19) is spatially distributed under the r-limited Delaunay graph $\mathcal{G}_{LD}$, for any agent configuration P.*

Proof. The control input u_i can be computed by the i-th agent with only the information available in the set $(V_i \cap \bar{B}(p_i, R))$, which defines the neighborhood relationship. Thus, each agent can compute the corresponding control input with only local information. □

Theorem 9 *The trajectories of the agents governed by the control law (19), starting from any initial condition $P(0) \in Q^N$, will asymptotically converge to the critical points of $\hat{\mathcal{H}}$.*

Proof. Consider $V(P) = -\hat{\mathcal{H}}(P)$, where $P = (p_1, p_2, ..., p_N)$ represents the configuration of N agents.

$$\dot{V}(P) = \tilde{C}_{(V_i \cap \bar{B}(p_i,R))} \times (-k_{prop})(p_i - C'_{(V_i \cap \bar{B}(p_i,R))}) \quad (20)$$

We observe that the conditions similar to Theorem 2 are valid here, with E itself being the largest invariant subset of E by the control law (14).

Thus, as before, by LaSalle's invariance principle [16, 17], the trajectories of the agents governed by control law (19), starting from any initial configuration $P(0) \in Q^N$, will asymptotically converge to the set N, the critical points of $\hat{\mathcal{H}}$, that is, the centroidal Voronoi partitions with respect to the density function as perceived by the sensors. □

5 Implementation Issues

Here we discuss some of the implementation issues involved in the proposed *deploy and search* strategy.

A single step of *deploy and search* strategy involves deploying the agents optimally, and then performing the search task within the respective Voronoi partitions. The deployment step can be implemented in *continuous time* (as given by control law (8)) or in *discrete time* (as in simulations carried out in this work). When the implementation is in discrete time, in each time step, the agents move towards the corresponding centroids and at the end of the deployment step, that is, when the agents are sufficiently close (as decided by the prescribed tolerance) to the centroids, the search task operation is carried out.

5.1 Discrete implementation

We convert the differential equation corresponding to the system dynamics (7) to a difference equation.

$$\frac{\Delta p_i}{\Delta t} = u_i \quad (21)$$

where Δt is the discrete time step.

Without loss of generality, we let $\Delta t = 1$ time unit, then (21) will be simplified as,

$$\Delta p_{ik} = u_{ik} \quad (22)$$

and the control law (8) takes the form,

$$u_{ik} = -k_{prop}(p_{ik} - C_{V_i k}) \quad (23)$$

where $k \in \mathbb{N}$ is the iteration count.

The control input u_{ik} is the desired speed of the i-th agent at the k-th time step, and the agent moves with this speed for Δt time units. With $\Delta t = 1$, u_{ik} acts as an increment on p_i per step. In other words $u_{ik} = \Delta p_i = p_{ik+1} - p_{ik}$. It can also be seen that, if $\Delta t = T$ time units, then the search task takes place after mT time units, where m is a non-zero integer, the number of time steps taken to achieve the optimal deployment. The process is illustrated in Figure 1. The consecutive search task is performed at a time interval of at most T time units. We define the *latency*, t_s, of the agents as the maximum time taken to acquire the information, process it, and successfully update the uncertainty density. T should be chosen to be greater than or equal to t_s.

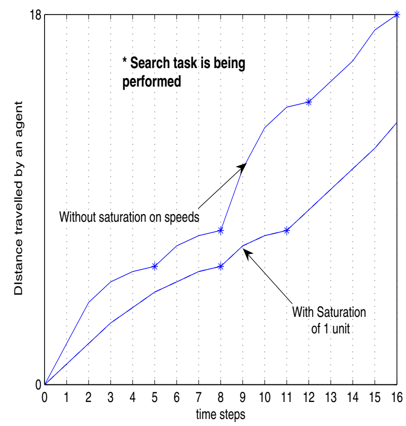

Figure 1. Illustration of the discrete time implementation of the *deploy and search* strategy. Deployment takes place according to control law (23) at every time step, and the search task takes place at the end of each deployment step indicated by '*'. With saturation of 1 unit on control input, that is, the speed of agents, the slope is restricted to a maximum of 1, and the deployment task might take longer time, delaying the search.

5.2 Effect of saturation

The control inputs given by control law (8) or (23) are nothing but the speeds of the agents. In practical implementation, it is likely that there will be constraint on the maximum speed of agents. Such a limit will appear as a saturation on the control input. In case of the *deploy and search* strategy, the effect of saturation on control input might lead to slower convergence of the deployment step. During the initial steps, it is likely that the control input provided by (8) can cross the saturation limit, whereas later, as the agents approach the centroids, the control input naturally reduces (as it is proportional to the distance between the agents and the respective centroids). Thus, effect of saturation is at most a possible increase in the time gap between consecutive search steps as illustrated in Figure 1.

5.3 Spatial distributedness

Here we discuss the implication of spatial distributedness of the proposed search strategy from a practical point of view. It has been shown that both the search strategies are spatially distributed over the Delaunay graph (Theorem 4). These results imply that all the agents need to do computations based on only local information, that is, by the knowledge about neighboring agents. The critical assumption here is the uncertainty density distribution within respective Voronoi partitions is available to all the agents. From a practical point of view this is true in the first search step, before the uncertainty density is updated, as the initial uncertainty density distribution is assumed to be known *a priori* to all the search agents. Once the uncertainty density is updated during the search task, the updated density information must be still available to all the agents, within the respective Voronoi partitions. It should be noted that the Voronoi partitions get updated as the agents move. If we let the agents communicate information about their positions in a spatially distributed manner (in the Delaunay graph), all the agents can compute the updated uncertainty density distribution as Delaunay graph is completely connected. Thus the proposed search strategy is truly spatially distributed.

In practical conditions, the agents can communicate with other agents only when they are within the limits of the sensor range. The Delaunay graph does not allow sensor range limitations to be incorporated. We need to use *r-limited Delaunay graph* or *r-Delaunay graph* to incorporate the sensor range limitations. It needs to be studied if the proposed search strategy is still spatially distributed in these graphs. In any case, the whole scenario changes with incorporation of sensor range limitations into the search strategy. The updating of uncertainty density will also be within the sensor range limits (in fact, it is within the intersection of sensor range limit disc and the corresponding Voronoi partitions). The centroid that can be computed will also be within the new restricted area. For an optimal deployment problem, from the perspective of sensor coverage, [10] shows that the corresponding control law is still spatially distributed (in r-limited Delaunay graph) and globally asymptotically stable. It needs to be examined if a similar situation occurs in control law (8) for the proposed search strategy.

6 Results and discussions

In this section we present results of some of the simulation experiments carried out to validate the search strategy

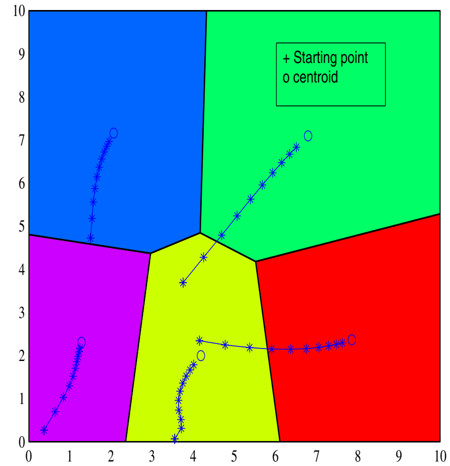

Figure 2. Trajectories of the agents and the final Voronoi diagram during initial deployment step with saturation of 1 unit.

proposed in this paper. A single step of deployment and search operation is referred to as one *step* or *iteration*. The simulation experiments were carried out using MATLAB (Ver 7.0). Multi-parametric tool box [5] was used for functions related to Voronoi partitions.

We have carried out a few sets of simulation experiments to validate the proposed search strategy. The parameters for these simulations were as follows

(i) Q is a square area in $\mathbb{R}^2$ with axes range of 0-10 units

(ii) Initial uncertainty density was a constant distribution of 0.75 over Q

(iii) A saturation on the speed of agents was fixed at 1 unit

(iv) $k_{prop} = 0.5$

(v) Sensor parameters were chosen as $k = 0.8$ and $\alpha = 0.1$

(vi) The iterations were terminated when the maximum density over Q reached below 0.05

Figure 2 shows the deployment step of *deploy and search* strategy for 5 agents. A saturation of 1 unit was imposed on the control law (8) to make the simulation more realistic. It is observed that the control law (8) does move the agents to the corresponding centroids successfully. In this specific case, the actual control input never crossed the saturation limit of 1 unit and hence trajectories with and without the saturation are identical.

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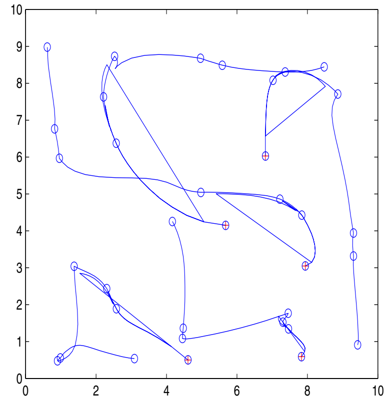

Figure 3. Trajectories of agents with N=5 and without sensor range limits ('o's indicate the end of each deployment step and the points in the space where the search task was being performed, the points marked '$\otimes$' indicate the starting locations of agents.)

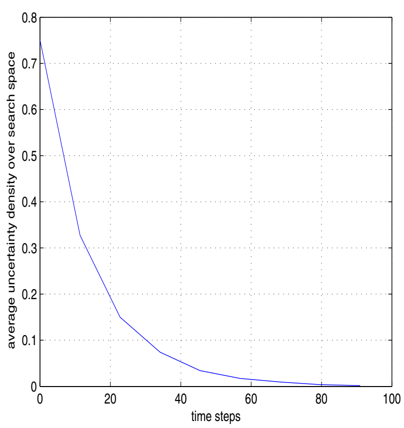

Figure 4. The reduction in uncertainty density distribution averaged over the search space as the iterations progress with N=5 and without sensor range limits

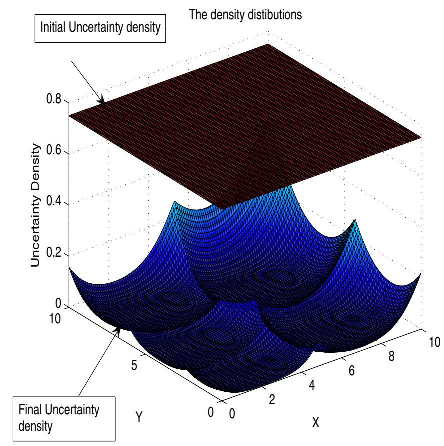

Figure 5. Initial and final uncertainty distribution for *Deploy and search*

strategy with $N = 5$ and without sensor range limits

Figure 3 shows the trajectories of 5 agents without any sensor range limitations. Figure 4 shows how the average uncertainty density changes with time steps. Figure 5 shows the initial and final uncertainty distributions over the search space Q. The final density distribution and the history of average density for all the simulation experiments are similar and hence not provided here.

Figures 6, 7, and 8 show the trajectories for N=5 and N=20, and with and without sensor range limitations.

Comparing Figure 3 with Figure 6, and Figure 7 with Figure 8, it can be observed that the sensor range limitation results in longer agent trajectories. This is due to reduced coverage by sensors due to the range limits.

If we compare the simulation results with $N = 5$ (Figures 3, 6) with those with $N = 20$ (Figure 7 and 8), it can be observed, that increase in number of agents lead to a smoother and somewhat shorter agent trajectories, though total trajectory length for all the agents, looks to be higher for the latter case.

Comparing Figures 3 and 7, it can be observed that, $N = 5$ leads to intersecting agent trajectories, whereas when $N = 20$, the strategy produces very less intersection among trajectories. In a search task, it is not desirable that an agent intersects its own trajectory, or intersects those of other agents, as it leads to duplication of effort. When the sensor range limits are imposed, (see Figures 6 and 8) the instances of intersection of trajectories increase.

In all cases (Figures 3, 6, 7, and 8) it can be observed that the agents move away from each other and cover the search space.

We have formulated the problem and shown that the *deploy and search* strategy reduces the uncertainty distribu-

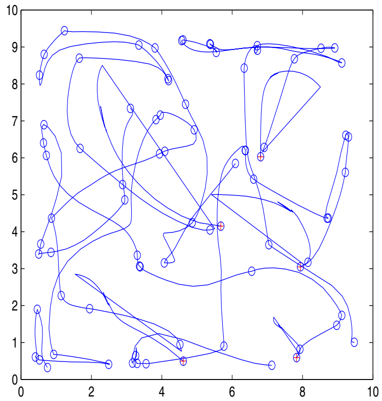

Figure 6. Trajectories of agents with N=5 and with sensor range limit of 2 units ('o's indicate the end of each deployment step and the points in the space where the search task was being performed, the points marked '$\otimes$' indicate the starting locations of agents.)

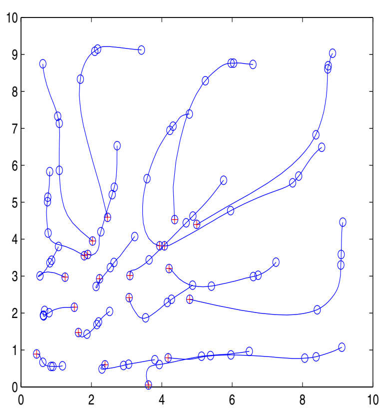

Figure 7. Trajectories of agents with N=20 ('o's indicate the end of each deployment step and the points in the space where the search task was being performed, the points marked '$\otimes$' indicate the starting locations of agents.)

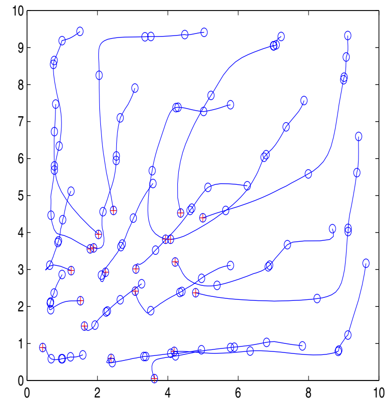

Figure 8. Trajectories of agents with N=20 and with sensor range limit of 2 units ('o's indicate the end of each deployment step and the points in the space where the search task was being performed, the points marked '$\otimes$' indicate the starting locations of agents.)

tion. The density given by (1) is a non-increasing function of time. The simulation results indicate that the proposed strategy leads to reduction of the density below the desired level. The simulations also indicate that the proposed strategy performs fairly well even with nominal sensor range limitations.

7 Conclusion

The problem of multi-agent search in an unknown environment with a known uncertainty probability distribution function is addressed as an extension and application of related concepts available in literature. We have analyzed the *deploy and search* strategy, where the agents first deploy themselves in the search space in an optimal way so as to maximize the one-step reduction in uncertainty. After deployment, the agents gather information in their respective Voronoi partitions and hence reduce the uncertainty. These iterations are continued till the uncertainty in the entire region is reduced to a required level. We have shown that the centroidal Voronoi configuration with respect to the density as perceived by the sensors are the critical points of the objective function. A control law was proposed, which moves the agents towards the respective centroids and shown to be globally asymptotically stable. It has been shown that the search strategy is *spatially distributed* over the Delaunay graph. We have proved that the *deploy and search* strategy proposed in this paper is able to reduce the average uncertainty density to arbitrarily low level. In addition, the proposed strategy has been analyzed for convergence and spatial distributedness under realistic constraints such as constraint on maximum speed of the agents, constant speed and limitation on sensor range. It has been shown that the control law moving agents towards the centroids of respective Voronoi partitions, with respect to the density as perceived by the sensors, are globally asymptotically stable and are also spatially distributed under these constraints.

Simulation experiments were carried out for different conditions and results of these experiments were discussed. The simulation results indicated that the proposed search strategy performs quite well even when the conditions deviated from the assumed ones such as sensor range limitations.

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