Stator flux based MRAS speed and stator resistance estimator for sensorless PMSM drive

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Abstract — **In this paper, a simple and robust speed and stator resistance estimator is proposed for sensorless permanent magnet synchronous motor drive to improve its performance at standstill and low speed regions. The speed and stator resistance estimator is formed using stator flux based model reference adaptive system, which yields fast transient response, reduces steady state error for low rotor speed variation and load disturbances. The proposed method improves the performance of the drive at low speed regions. The simulations are carried out through MATLAB/Simulink environment and results show the effectiveness of the sensorless PMSM drive. The maximum estimated rotor speed and position error are found to be are ±3 rad/s, ±1°respectively, during low speed operation at transient and steady state conditions.**
Keywords: permanent magnet synchronous motor (PMSM), model reference adaptive system (MRAS), sensorless speed and position estimation, field oriented control (FOC).

I. INTRODUCTION

In recent years, PMSM drives are the first choice in industrial applications and home appliances such as fans, air conditioners, washing machines and dishwashers. The field oriented control of PMSM drive [1] has drawn attention from both industry and academia due to its high dynamic performance. Sensorless rotor speed and position estimation schemes are broadly grouped as: fundamental excitation methods, saliency and signal injection methods. The fundamental excitation methods can be grouped into open loop and closed loop estimators. The closed loop estimation schemes are divided into two categories: state observer based methods and MRAS. State observer based methods are further classified into: stator flux based observer methods [2-3], sliding mode observer methods [4-5], Extended Kalman filter method [6]. Observer based methods, require knowledge of machine parameters. At standstill or low speed regions, the speed and position estimation performance are affected by noise, error and delay. Though Extended Kalman filter and sliding mode observer perform well at standstill and low speed operation, but they suffer from high computational complexity and chattering phenomenon. MRAS based methods discussed in [6-8] for rotor speed and position estimation have drawn attention due to their simplicity. MRAS speed and position estimators based on stator current, stator flux, real power and reactive power have been proposed for vector control of PMSM drive. The main drawback of these techniques is machine parameter sensitivity which constraints its performance at standstill and low speed regions. The stator resistance value varies from nominal value due to ageing and thermal variations in the motor. So, stator resistance estimation method is proposed to improve robustness of PMSM drive. There are several rotor speed/ position schemes are proposed in the literature [2-8]. The MRAS based speed and position estimation methods are simple, robust and requires less computational time [9], [10]. In this paper, stator flux based MRAS speed and resistance (stator) estimator is presented to improve the robustness of speed and position estimation at low speed and standstill operation. The proposed method has significant advantage compared to previous speed estimation methods [2-8] and the proposed method is suitable for low cost and industrial applications.

In this paper, stator flux based MRAS speed and stator resistance estimator is presented to improve the robustness of speed and position estimation at low speed and standstill operation. This paper is organized as follows, the proposed stator flux based MRAS speed and stator resistance estimator is elaborated in section 2 and 3. Simulation results are demonstrated in section 4.

II. STATOR FLUX BASED MRAS SPEED ESTIMATOR

Fig. 1. shows the structure of stator flux based MRAS speed estimator. In this work, the rotor speed is estimated by differences in stator flux. Real system is considered as reference model and it is independent with respect to estimated quantity which provides ψ_{ds} and ψ_{qs}. The mathematical flux model is considered as the adjustable model and it includes estimated quantity, which provides $\hat{\psi}_{ds}$ and $\hat{\psi}_{qs}$. The error signal is obtained from comparing reference and estimated stator flux and it is processed through the PI controller. The stator flux error signal is used to estimate the rotor speed. The Structure of sensorless FOC of PMSM drive with proposed MRAS speed and stator resistance estimator is depicted in Fig. 2.

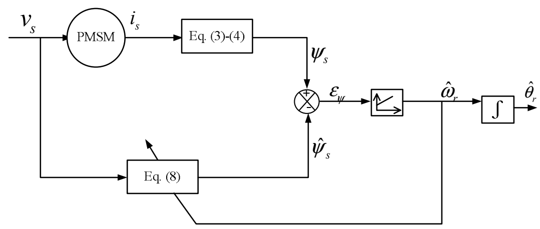

Fig. 1. Structure of the stator flux based MRAS speed estimator.

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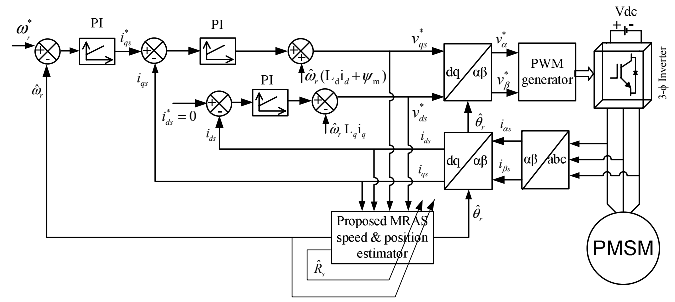

Fig. 2. Structure of sensorless FOC of PMSM drive with proposed MRAS speed and stator resistance estimator.

The stator *d-q* voltages in terms of machine variables are expressed as [1].

$$v_{qs} = \hat{R}_s i_{qs} + \omega_r \psi_{ds} + p\psi_{qs} \tag{1}$$

$$v_{ds} = \hat{R}_s i_{ds} - \omega_r \psi_{qs} + p\psi_{ds} \tag{2}$$

Stator flux linkages can be expressed as

$$\psi_{ds} = L_d i_{ds} + \psi_m \tag{3}$$

$$\psi_{qs} = L_q i_{qs} \tag{4}$$

by substituting (3) and (4) in (1) and (2), the stator flux can be written in matrix form as given

$$\frac{d}{dt}\begin{bmatrix}\psi_{ds}\\ \psi_{qs}\end{bmatrix} = \begin{bmatrix}-\frac{\hat{R}_s}{L_d} & \omega_r\\ -\omega_r & -\frac{\hat{R}_s}{L_q}\end{bmatrix}\begin{bmatrix}\psi_{ds}\\ \psi_{qs}\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}V_{ds} + \frac{\psi_m \hat{R}_s}{L_d}\\ V_{qs}\end{bmatrix} \tag{5}$$

The above (5) is considered as reference model, which is further simplified and given as below

$$\dot{\psi} = A_1\psi + Bu \tag{6}$$

$$\psi = C\psi \tag{7}$$

Where

$$A_1 = \begin{bmatrix}-\frac{\hat{R}_s}{L_d} & \omega_r\\ -\omega_r & -\frac{\hat{R}_s}{L_q}\end{bmatrix}, \psi = \begin{bmatrix}\psi_{ds} & \psi_{qs}\end{bmatrix}^T, B = C = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, u = \begin{bmatrix}V_{ds} + \frac{\psi_m \hat{R}_s}{L_d}\\ V_{qs}\end{bmatrix}$$

In (6) the parameters stator flux and rotor speed are replaced with estimated quantities which forms adjustable model and given as

$$\dot{\hat{\psi}} = A_2\hat{\psi} + Bu \tag{8}$$

$$\hat{\psi} = C\hat{\psi} \tag{9}$$

where $A_2 = \begin{bmatrix}-\hat{R}_s/L_d & \hat{\omega}_r\\ -\hat{\omega}_r & -\hat{R}_s/L_q\end{bmatrix}, \hat{\psi} = \begin{bmatrix}\hat{\psi}_{ds} & \hat{\psi}_{qs}\end{bmatrix}^T$

The stator flux error is obtained between reference model and adjustable model outputs (i.e. reference stator flux and estimated stator flux).

$$\varepsilon_{ds} = \psi_{ds} - \hat{\psi}_{ds}, \ \varepsilon_{qs} = \psi_{qs} - \hat{\psi}_{qs} \tag{10}$$

subtracting (8) from (6) and written as

$$\frac{d\varepsilon_{ds}}{dt} = \frac{-R_s}{L_d}\varepsilon_{ds} + \omega_r\psi_{qs} - \hat{\omega}_r\psi_{qs} \tag{11}$$

$$\frac{d\varepsilon_{qs}}{dt} = \frac{-R_s}{L_q}\varepsilon_{qs} - \omega_r\psi_{ds} + \hat{\omega}_r\psi_{ds} \tag{12}$$

Adding and subtracting of the terms $\hat{\omega}_r\psi_{qs}$ in (11) and $\hat{\omega}_r\psi_{ds}$ in (12). The state error can be written as follow

$$\frac{d\varepsilon_{ds}}{dt} = \frac{-\hat{R}_s}{L_d}\varepsilon_{ds} + \hat{\omega}_r\varepsilon_{qs} + \psi_{qs}(\omega_r - \hat{\omega}_r) \tag{13}$$

$$\frac{d\varepsilon_{qs}}{dt} = \frac{-\hat{R}_s}{L_q}\varepsilon_{qs} - \hat{\omega}_r\varepsilon_{ds} - \psi_{ds}(\omega_r - \hat{\omega}_r) \tag{14}$$

Eq. (13) and (14) can be written in matrix form as below

$$\frac{d}{dt}\begin{bmatrix}\varepsilon_{ds}\\ \varepsilon_{qs}\end{bmatrix} = \begin{bmatrix}\frac{-\hat{R}_s}{L_d} & \hat{\omega}_r\\ -\hat{\omega}_r & \frac{-\hat{R}_s}{L_q}\end{bmatrix}\begin{bmatrix}\varepsilon_{ds}\\ \varepsilon_{qs}\end{bmatrix} + \begin{bmatrix}\psi_{qs}\\ -\psi_{ds}\end{bmatrix}(\omega_r - \hat{\omega}_r) \tag{15}$$

State error model is simplified as follows

$$\dot{\varepsilon}_\psi = A_2\varepsilon_\psi + W_1 \tag{16}$$

where $\varepsilon_\psi = \begin{bmatrix}\varepsilon_{ds} & \varepsilon_{qs}\end{bmatrix}^T$, $W_1 = \begin{bmatrix}\psi_{qs}\\ -\psi_{ds}\end{bmatrix}(\omega_r - \hat{\omega}_r)$

To ensure the system is stable, which requires the state error (ε) to be near to zero. The stability of the system is analyzed using Popov super hyperstability theorem. The state error depends on two criterions, where the first is linear time variant matrix, $H(s) = (sI - A_2)^{-1}$ should be a strictly positive matrix and the second is nonlinear term, $\eta(0, t_0) = \int_0^{t_0}\left[\varepsilon_\psi\right]^T\left[W_1\right]dt \geq -\gamma_0^2, \ \forall t_0 \geq 0$ where γ_0 can be finite positive number.

The above expression can be modified using W_1

$$\int_0^{t_0}\left[\varepsilon_{ds}\psi_{qs} - \varepsilon_{qs}\psi_{ds}\right](\omega_r - \hat{\omega}_r)\,dt \geq -\gamma_0^2 \tag{17}$$

Finally, the estimated rotor speed is obtained using stator flux error which is given below

$$\hat{\omega} = \int_0^t k_1\left(\varepsilon_{ds}\psi_{qs} - \varepsilon_{qs}\psi_{ds}\right)dt + k_2\left(\varepsilon_{ds}\psi_{qs} - \varepsilon_{qs}\psi_{ds}\right) \tag{18}$$

where $k_1, k_2 \geq 0$

The estimated rotor position is achieved by integrating the estimated speed.

$$\hat{\theta} = \int \hat{\omega}\,dt \tag{19}$$

Fig. 3 shows the pole-zero plot for the transfer matrix *H(s)* for the speed range ($\hat{\omega}_r = 0 \rightarrow \pm 180$ rad/s). It is observed that, all

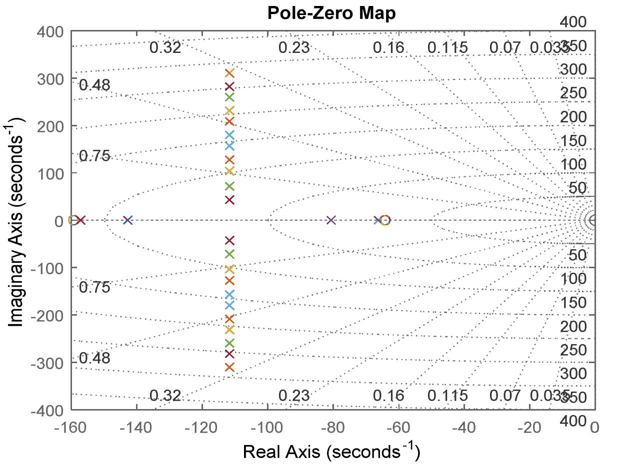

Fig. 3. Pole placement for the transfer matrix H(s), $\hat{\omega}_r = 0 \rightarrow \pm 180$ rad/s.

the poles are located in the left half of the s plane. So, it confirms that (H(s)) is completely positive matrix.

2.1 Stability analysis of the proposed MRAS speed estimator

To validate the dynamic response of the stator flux based MRAS speed estimator stability analysis is performed by linearizing the stator flux equation (6) and (7) around a stable point ψ_{0}, which is as follows:

$$\Delta\dot{\psi} = A_1\Delta\psi + \Delta A_1\psi_0 \tag{20}$$

$$\Delta\psi = C\Delta\psi \tag{21}$$

$$\Delta\psi = C(\mathrm{sI}-\mathrm{A}_1)^{-1}\Delta A_1\psi_0 \tag{22}$$

where $\psi_0 = [\psi_{sd0} \quad \psi_{sq0}]^T$

now consider small variation in speed, then ΔA_1 can be expressed as

$$\Delta A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\Delta\omega_r \tag{23}$$

by substituting (23) into (22), the $\Delta\psi$ can be expressed as

$$\begin{bmatrix} \Delta\psi_{sd} \\ \Delta\psi_{sq} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}(\mathrm{sI}-\mathrm{A}_1)^{-1}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\Delta\omega_r\begin{bmatrix} \psi_{sd0} \\ \psi_{sq0} \end{bmatrix} \tag{24}$$

From (24), the transfer function of $\left(\dfrac{\Delta\psi_{sd}}{\Delta\omega_r}\right)$ and $\left(\dfrac{\Delta\psi_{sq}}{\Delta\omega_r}\right)$ can be expressed as

$$\frac{\Delta\psi_{ds}}{\Delta\omega_r} = \frac{-\omega_r\psi_{ds0} + (\mathrm{s}+\hat{R}_s/L_q)\psi_{qs0}}{(\mathrm{s}+\hat{R}_s/L_q)*(\mathrm{s}+\hat{R}_s/L_d)+\omega_r^2} \tag{25}$$

$$\frac{\Delta\psi_{qs}}{\Delta\omega_r} = \frac{-(\mathrm{s}+\hat{R}_s/L_d)\psi_{ds0} - \hat{\omega}_r\psi_{qs0}}{(\mathrm{s}+\hat{R}_s/L_q)*(\mathrm{s}+\hat{R}_s/L_d)+\omega_r^2} \tag{26}$$

By linearizing the stator flux error (17) and dividing by $\Delta\omega_r$ yields the transfer function between $\Delta\varepsilon_\psi$ and $\Delta\omega_r$

$$\frac{\varepsilon_\psi}{\Delta\omega_r} = \frac{\Delta\psi_{ds}*\psi_{qs}}{\Delta\omega_r} - \frac{\Delta\psi_{qs}*\psi_{ds}}{\Delta\omega_r} \tag{27}$$

Substituting (25)-(26) in (27), the stator flux error transfer function can be expressed as

$$\frac{\varepsilon_\psi}{\Delta\omega_r} = G(s) = \frac{\left(-\omega_r\psi_{ds0} + (\mathrm{s}+\hat{R}_s/L_q)\psi_{qs0}\right)*\psi_{qs}}{(\mathrm{s}+\hat{R}_s/L_q)*(\mathrm{s}+\hat{R}_s/L_d)+\omega_r^2} + \frac{\left((\mathrm{s}+\hat{R}_s/L_d)\psi_{ds0} + \hat{\omega}_r\psi_{qs0}\right)*\psi_{ds}}{(\mathrm{s}+\hat{R}_s/L_q)*(\mathrm{s}+\hat{R}_s/L_d)+\omega_r^2} \tag{28}$$

finally, the closed loop transfer function of proposed MRAS speed estimator is given as

$$\frac{\omega_r}{\hat{\omega}_r} = \frac{G(s)(\mathrm{K_{pMRAS}} + \frac{\mathrm{K_{iMRAS}}}{s})}{1+G(s)(\mathrm{K_{pMRAS}} + \frac{\mathrm{K_{iMRAS}}}{s})} \tag{29}$$

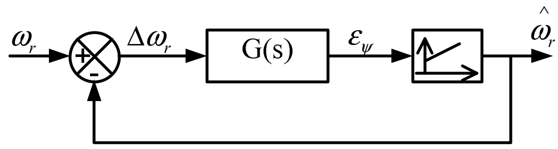

Fig. 4. Closed representation of MRAS speed and position estimator

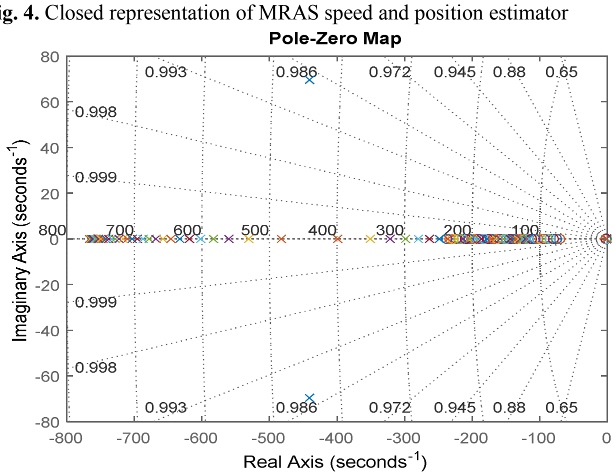

Fig. 5. Loci for the closed loop transfer function of proposed MRAS speed estimator, $\hat{\omega}_r = 0 \rightarrow \pm 180$rad/s

Fig. 4 shows the closed loop representation of proposed MRAS rotor speed estimator. The stability of the proposed system is analyzed through pole placement for the closed loop transfer function of proposed MRAS speed estimator. Fig 5 shows loci of the closed loop poles and zeros for the rated speed range ($\hat{\omega}_r = 0 \rightarrow \pm 180$ rad/s). It is observed from fig. 5 that all the poles are located in the left half of the s plane and it shows the proposed MRAS speed estimator is stable for motoring and speed reversal operation for entire operating region.

III. STATOR FLUX BASED MRAS STATOR RESISTANCE ESTIMATOR

Fig. 6. shows the structure of stator flux based MRAS stator resistance estimator. In this work, the stator resistance is estimated by differences in stator flux. Real system is considered as reference model and it is independent with respect to stator resistance which provides ψ_{ds1} and ψ_{qs1}. The mathematical flux model is considered as the adjustable model and it includes stator resistance, which provides $\hat{\psi}_{ds1}$ and $\hat{\psi}_{qs1}$. The error signal is obtained from comparing reference and

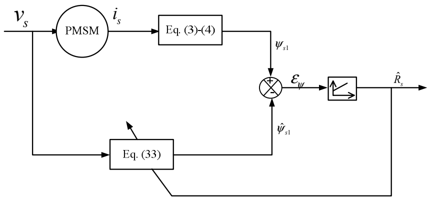

Fig. 6. Structure of the stator flux based MRAS stator resistance estimator.

estimated stator flux and it is processed through the PI controller. The stator flux error signal is used to estimate the stator resistance. The stator flux can be written in matrix form given as

$$\frac{d}{dt}\begin{bmatrix}\psi_{ds1}\\ \psi_{qs1}\end{bmatrix}=\begin{bmatrix}-\frac{R_s}{L_d} & \hat{\omega}_r\\ -\hat{\omega}_r & -\frac{R_s}{L_q}\end{bmatrix}\begin{bmatrix}\psi_{ds1}\\ \psi_{qs1}\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}V_{ds}+\frac{\psi_m R_s}{L_d}\\ V_{qs}\end{bmatrix} \quad (30)$$

The above (30) is considered as reference model, which is further simplified and given as below

$$\dot{\psi}_1 = A_3\psi_1 + Bu_1 \quad (31)$$

$$\psi_1 = C\psi_1 \quad (32)$$

where $A_3=\begin{bmatrix}-\frac{R_s}{L_d} & \hat{\omega}_r\\ -\hat{\omega}_r & -\frac{R_s}{L_q}\end{bmatrix}$, $\psi_1=\begin{bmatrix}\psi_{ds1} & \psi_{qs1}\end{bmatrix}^T$, $u_1=\begin{bmatrix}V_{ds}+\frac{\psi_m R_s}{L_d}\\ V_{qs}\end{bmatrix}$

In (31) the parameters stator flux and resistance are replaced with estimated quantities which forms adjustable model and given as

$$\dot{\hat{\psi}}_1 = A_2\hat{\psi}_1 + Bu \quad (33)$$

$$\hat{\psi}_1 = C\hat{\psi}_1 \quad (34)$$

where $\hat{\psi}_1=\begin{bmatrix}\hat{\psi}_{ds1} & \hat{\psi}_{qs1}\end{bmatrix}^T$

The stator flux error is obtained between reference model and adjustable model outputs (i.e. reference stator flux and estimated stator flux).

$$\varepsilon_{ds1}=\psi_{ds1}-\hat{\psi}_{ds1},\ \varepsilon_{qs1}=\psi_{qs1}-\hat{\psi}_{qs1} \quad (35)$$

Subtracting (33) from (31) and written as

$$\frac{d\varepsilon_{ds1}}{dt}=\frac{-R_s\psi_{ds1}}{L_d}+\hat{\omega}_r\varepsilon_{qs1}+\frac{R_s\psi_m}{L_d}+\frac{\hat{R}_s\hat{\psi}_{ds1}}{L_d}-\frac{\hat{R}_s\psi_m}{L_d} \quad (36)$$

$$\frac{d\varepsilon_{qs1}}{dt}=-\hat{\omega}_r\varepsilon_{ds}-\frac{R_s\psi_{qs}}{L_q}+\frac{\hat{R}_s\hat{\psi}_{qs}}{L_q} \quad (37)$$

Adding and subtracting of the terms $\frac{\hat{R}_s\psi_{ds1}}{L_d}$ in (36) and $\frac{\hat{R}_s\psi_{qs1}}{L_q}$ in (37). The state error can be written as follow

$$\frac{d\varepsilon_{ds1}}{dt}=\frac{-\hat{R}_s}{L_d}\varepsilon_{ds1}+\hat{\omega}_r\varepsilon_{qs1}+\left[\frac{-\psi_{ds1}+\psi_m}{L_d}\right](R_s-\hat{R}_s) \quad (38)$$

$$\frac{d\varepsilon_{qs1}}{dt}=\frac{-\hat{R}_s}{L_q}\varepsilon_{qs1}-\hat{\omega}_r\varepsilon_{ds1}-\frac{\psi_{qs1}}{L_q}(\mathrm{R}_s-\hat{R}_s) \quad (39)$$

Eq. (38) and (39) can be written in matrix form as given

$$\frac{d}{dt}\begin{bmatrix}\varepsilon_{ds1}\\ \varepsilon_{qs1}\end{bmatrix}=\begin{bmatrix}\frac{-\hat{R}_s}{L_d} & \hat{\omega}_r\\ -\hat{\omega}_r & \frac{-\hat{R}_s}{L_q}\end{bmatrix}\begin{bmatrix}\varepsilon_{ds1}\\ \varepsilon_{qs1}\end{bmatrix}+\begin{bmatrix}\frac{-\psi_{ds1}+\psi_m}{L_d}\\ \frac{-\psi_{qs1}}{L_q}\end{bmatrix}(R_s-\hat{R}_s) \quad (40)$$

The above (40) is stator flux error, which is further simplified and given as below

$$\dot{\varepsilon}_{\psi1}=A_2\varepsilon_{\psi1}+W_2 \quad (41)$$

where $\varepsilon_{\psi1}=\begin{bmatrix}\varepsilon_{ds1} & \varepsilon_{qs1}\end{bmatrix}^T$, $W_2=\begin{bmatrix}\frac{-\psi_{ds1}+\psi_m}{L_d}\\ \frac{-\psi_{qs1}}{L_q}\end{bmatrix}(R_s-\hat{R}_s)$

To ensure the system stability, it requires that state error ($\varepsilon_{\psi1}$) should be near to zero. The stability of the system is analyzed using Popov super hyperstability theorem, state error depends on two criterions. The first is linear time variant matrix, $H_1(s)=(sI-A_2)^{-1}$ should be a strictly positive matrix and the second is nonlinear term, $\eta(0,t_0)=\int_0^{t_0}\left[\varepsilon_{\psi1}\right]^T\left[W_2\right]dt\geq-\gamma_0^2$, $\forall t_0\geq 0$ where γ_0 can be finite positive number.

The above expression can be modified by using W_2

$$\int_0^{t_0}\left[\left(\frac{-\psi_{ds1}+\psi_m}{L_d}\right)\varepsilon_{ds1}-\left(\frac{\psi_{qs1}}{L_q}\right)\varepsilon_{qs1}\right](R_s-\hat{R}_s)\,dt\geq-\gamma_0^2 \quad (42)$$

Fig. 7 shows the pole-zero plot for the transfer matrix *$H_1(s)$* for the stator resistance variation from -50% to +50% with respect to nominal value for the speed range ($\hat{\omega}_r$ = 0→±180 rad/s). It is observed that all the poles are located in the left-half of the s plane. So, it confirms that ($H_1(s)$) is completely positive matrix.

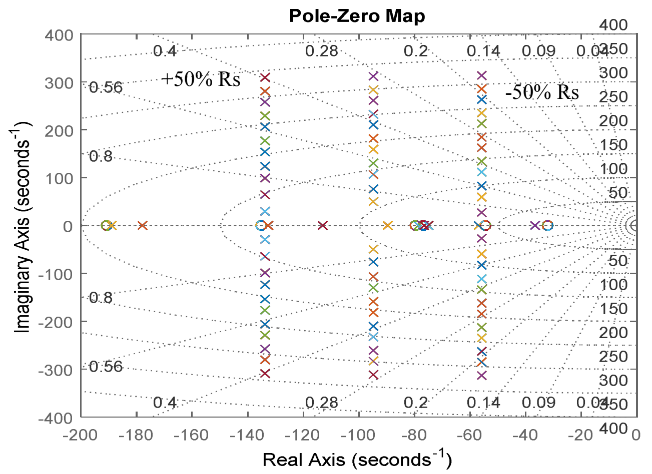

Fig. 7. Pole placement for the closed loop transfer function of proposed MRAS stator resistance estimation, $\hat{\omega}_r = 0 \rightarrow \pm 180$ rad/s, r_s= -50% to +50% of their nominal value.

The estimated stator resistance can be obtained as follows

$$\hat{R}_s = \int_0^t k_1\left[\left(\frac{-\psi_{ds1}+\psi_m}{L_d}\right)\varepsilon_{ds1} - \left(\frac{\psi_{qs1}}{L_q}\right)\varepsilon_{qs1}\right]dt + k_2\left[\left(\frac{-\psi_{ds1}+\psi_m}{L_d}\right)\varepsilon_{ds1} - \left(\frac{\psi_{qs1}}{L_q}\right)\varepsilon_{qs1}\right] \tag{43}$$

where $k_1, k_2 \geq 0$

In the above equation ψ_{ds1} and ψ_{qs1} can be calculated through the reference model, $\hat{\psi}_{ds1}$ and $\hat{\psi}_{qs1}$ can be obtained through the adjustable model.

3.1 *Stability analysis of the proposed MRAS stator resistance estimator*

To validate the dynamic response of the stator flux based MRAS resistance estimator stability analysis is performed by linearizing the stator flux equation (31) and (32) around a stable point ψ_{01}, which is as follows:

$$\Delta\dot{\psi}_1 = A_3\Delta\psi_1 + \Delta A_3\psi_{01} \tag{44}$$

$$\Delta\psi_1 = C\Delta\psi_1 \tag{45}$$

$$\Delta\psi_1 = C(\mathrm{sI} - \mathrm{A}_3)^{-1}\Delta A_3\psi_{01} \tag{46}$$

Where $\psi_{01} = [\psi_{sd01} \quad \psi_{sq01}]^T$

now consider small variation in stator resistance, then ΔA_3 can be expressed as

$$\Delta A_3 = \begin{bmatrix} \frac{-1}{L_d} & 0 \\ 0 & \frac{-1}{L_q} \end{bmatrix}\Delta R_s \tag{47}$$

By substituting (47) into (46), the Δy can be expressed as

$$\begin{bmatrix} \Delta\psi_{sd1} \\ \Delta\psi_{sq1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}(\mathrm{sI} - \mathrm{A}_3)^{-1}\begin{bmatrix} -\frac{1}{L_d} & 0 \\ 0 & -\frac{1}{L_q} \end{bmatrix}\Delta R_s\begin{bmatrix} \psi_{sd01} \\ \psi_{sq01} \end{bmatrix} \tag{48}$$

From (48), the transfer function of $\left(\frac{\Delta\psi_{sd1}}{\Delta R_s}\right)$ and $\left(\frac{\Delta\psi_{sq1}}{\Delta R_s}\right)$ can be expressed as

$$\frac{\Delta\psi_{ds1}}{\Delta R_s} = \frac{-\left(\mathrm{s}+R_s/L_q\right)\left(\psi_{ds01}/L_d\right) - \left(\hat{\omega}_r * \psi_{qs01}/L_q\right)}{(\mathrm{s}+R_s/L_q)*(\mathrm{s}+R_s/L_d)+\hat{\omega}_r^2} \tag{49}$$

$$\frac{\Delta\psi_{qs1}}{\Delta R_s} = \frac{\left(\hat{\omega}_r/L_d\right)\left(\psi_{ds01}\right) - \left(\mathrm{s}+R_s/L_d\right)*\left(\psi_{qs01}/L_q\right)}{(\mathrm{s}+R_s/L_q)*(\mathrm{s}+R_s/L_d)+\hat{\omega}_r^2} \tag{50}$$

By linearizing the stator flux error (42) and dividing by ΔR_s yields the transfer function between $\Delta\varepsilon_\psi$ and ΔR_s

$$\frac{\varepsilon_{\psi1}}{\Delta R_s} = \left[\left(\frac{-\psi_{ds1}+\psi_m}{L_d}\right)\frac{\Delta\psi_{ds1}}{\Delta R_s} - \left(\frac{\psi_{qs1}}{L_q}\right)\frac{\Delta\psi_{qs1}}{\Delta R_s}\right] \tag{51}$$

Substitute (49)-(50) into (51), the stator flux error transfer function can be expressed as

$$\frac{\varepsilon_\psi}{\Delta R_s} = G_1(\mathrm{s}) = \frac{\varepsilon_{\psi1}}{\Delta R_s} = \left(\frac{-\psi_{ds1}+\psi_m}{L_d}\right)\left(\frac{-\left(\mathrm{s}+R_s/L_q\right)\left(\psi_{ds01}/L_d\right) - \left(\hat{\omega}_r * \psi_{qs01}/L_q\right)}{(\mathrm{s}+R_s/L_q)*(\mathrm{s}+R_s/L_d)+\hat{\omega}_r^2}\right) - \left(\frac{\psi_{qs1}}{L_q}\right)\left(\frac{\left(\hat{\omega}_r/L_d\right)\left(\psi_{ds01}\right) - \left(\mathrm{s}+R_s/L_d\right)*\left(\psi_{qs01}/L_q\right)}{(\mathrm{s}+R_s/L_q)*(\mathrm{s}+R_s/L_d)+\hat{\omega}_r^2}\right) \tag{52}$$

The closed loop representation of proposed MRAS stator resistance estimator

$$\frac{R_s}{\hat{R}_s} = \frac{G_1(\mathrm{s})(\mathrm{K_{p,res}} + \frac{\mathrm{K_{i,res}}}{s})}{1 + G_1(\mathrm{s})(\mathrm{K_{p,res}} + \frac{\mathrm{K_{i,res}}}{s})} \tag{53}$$

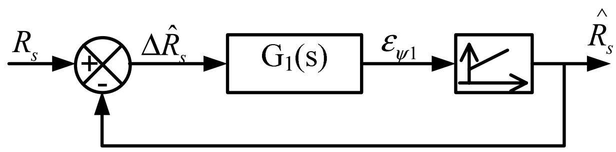

Fig. 8. Closed representation of MRAS stator resistance estimator

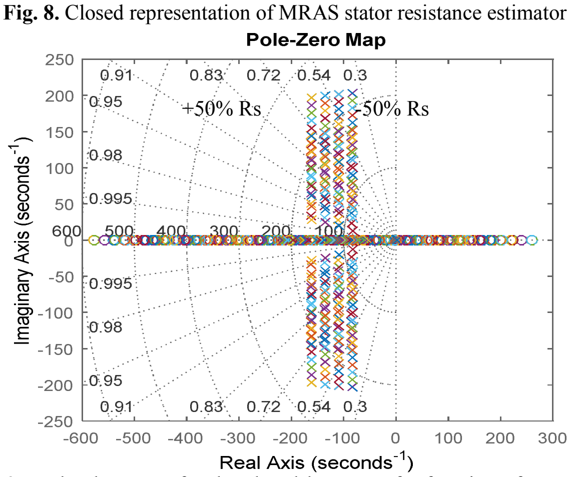

Fig. 9. Pole placement for the closed loop transfer function of proposed MRAS stator resistance estimator for the stator resistance variation from -50% to +50% with respect to nominal value for the speed range ($\hat{\omega}_r$ = 0→±180 rad/s).

Fig. 8 shows the closed loop representation of proposed stator flux based MRAS stator resistance estimator. The stability of the proposed system is analyzed through pole placement for the closed loop transfer function of proposed MRAS based stator resistance estimator. Fig 9 shows loci of the closed loop poles and zeros where stator resistance is varied from -50 % to +50% with respect to nominal value for the speed range ($\hat{\omega}_r$ = 0→±180 rad/s rad/s). It is observed that all the poles are located in the left-hand side of the s plane and it shows the proposed stator flux based MRAS stator resistance estimator is stable for motoring and speed reversal operation for entire operating region.

IV. SIMULATION RESULTS AND DISCUSSIONS

The proposed stator flux based MRAS speed and stator resistance estimator for PMSM drive is simulated in the

MATLAB / SIMULINK environment. The proposed system is investigated under the following condition: low speed variation with stator resistance variation. The parameters of the PMSM used to validate the proposed system are given in Table I.

A. Low speed variation

Fig.10 shows the speed is gradually varied between 0 to 20 rad/s, -20, 10 and 30rad/s at rated load. The estimated rotor speed follows the measured speed during speed reversal and zero speed operation satisfactorily, as shown in Fig.10a. The measured and estimated rotor position are shown in Fig. 10b. From this analysis, it is clear that estimated rotor position is virtually similar to the measured rotor position. The estimated rotor speed and position error are ±3 rad/s, ±1° respectively as shown in Fig.10c and d. The estimated stator resistance is shown in Fig. 10e. It is observed that, the proposed speed and stator resistance estimator perfoms well at low speed regions with 50% of increase in stator resistance.

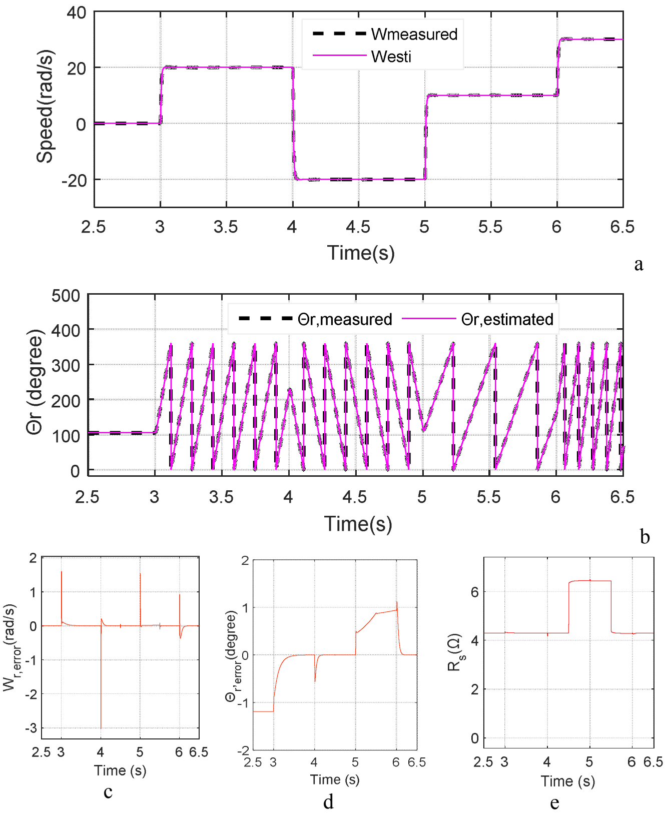

Fig. 10. Simulation results of ramp type rotor speed response
a. measured and estimated rotor speed
b. measured and estimated rotor position
c. speed estimation error
d. position estimation error
e. stator resistance

V. CONCLUSIONS

A simple and robust rotor speed and stator resistance estimation for sensorless PMSM drive is proposed and investigated in this paper. Stator flux based MRAS speed and stator resistance estimator is designed and implemented. The stability of the proposed MRAS based speed and stator resistance estimator method is confirmed through small signal analysis and proposed method performs well at standstill and low speed region. Simulation results demonstrate the effectiveness of the sensorless speed and stator resistance estimation of FOC with PMSM drive.

APPENDIX

TABLE I. PMSM RATING AND PARAMETERS
Stator: 3ph, 4 pole, 220v, 1.2hp, 1700 rpm

Parameter	Measured value in SI units
R_s	4.2 Ω
L_d	30 mH
L_q	65 mH
λ_m	0.272 Wb
J	0.00018 kg m^2

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