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THE EFFECT OF THE THICKNESS OF THE POROUS MATERIAL ON THE PARALLEL PLATE CHANNEL FLOW WHEN THE WALLS ARE PROVIDED WITH NON-ERODIBLE POROUS LINING

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Abstract

Flow through a channel whose walls are lined with non-erodible porous material is investigated using Beavers and Joseph slip boundary condition. It is shown that the effect of porous lining is to increase the mass flow rate and to decrease the friction factor.

Nomenclature

u	streamwise velocity in Zone 1 (of Fig. 1 and Fig. 2)
h	height of the channel
h'	thickness of the porous lining
Q	Darcy velocity
p	pressure
μ	viscosity
K	absolute permeability of the material
α	slip parameter
ρ	density
λ	resistance coefficient
R	Reynolds number
u_B	slip velocity at the nominal surface
$\bar{u}$	the average velocity in Zone 1
M_1	non-dimensional mass flow rate in Zone 1 (Fig. 1)

M_2 non-dimensional mass flow rate in Zone 2 (Fig. 1)
M^* non-dimensional mass flow rate in the channel without any porous lining
m_1 non-dimensional mass flow rate in Zone 1 (Fig. 4)
m_2 non-dimensional mass flow rate in Zone 2 (Fig. 4)
m_3 non-dimensional mass flow rate in Zone 3 (Fig. 4)
$(\lambda R)_1$ the λR-product for flow in Zone 1 (Fig. 1)
$(\lambda R)_2$ the λR-product for flow in Zone 1 (Fig. 4)
$(\lambda R)_*$ the λR-product for flow in the channel of height h without porous lining

§ 1. Introduction

In recent years considerable interest has been evinced in the study of flow past porous media because of its application in industrial, biophysical and hydrological problems.

In the study of flow past a porous material it is customary to use the no-slip boundary condition at the porous surface where the effect of porosity is taken care of by the continuity of the normal component of velocity. However, Beavers and Joseph [1] have investigated, for the first time, this class of flows past a naturally permeable bed with slip at the nominal surface (here after called BJ boundary condition). Subsequently Beavers et al [2], Taylor [3], Richardson [4] and Rajasekhara [5] have confirmed, experimentally, the BJ boundary condition. A rigorous theoretical justification for BJ boundary condition has been given by Saffman [6]. Recently, this problem has been extended to flows of electrically conducting fluids by Rudraiah et al [7] and Chandrasekhara [8]. To bridge the gap between the theoretical and experimental work of Rajasekhara [5], recently Veerabhadraiah and Rudraiah [9] have considered the combined forced and free convection problem and they have shown that even in horizontal flow the gravity plays a significant part. In all the above investigations the thickness of the permeable bed has not directly entered the analysis, because, the situation considered in each of the above cases is that of the flow past a naturally permeable bed.

In practical problems, involving the flow past a porous lining, it is necessary to involve directly the thickness of the porous lining to achieve increase in mass flow rate. Therefore, the object of this paper is to study the effect of the thickness of the porous lining on the parallel plate channel flow.

In practice, the channel may be bounded on one side by a rigid plate and the other may be lined with non-erodible porous material, or both sides may be lined with porous material. Both these situations are considered in this paper, the former in § 2 and the latter in § 3. In § 4, we compute Resistance coefficients which will be useful for the experimental determination of the breakdown of laminar flow. § 5 is devoted to the discussion of results. We find that the thickness of the porous lining has considerable influence on the mass flow rate and hence on the friction factor. The results of this paper have paved the way for further work particularly to study the stability of this system.

§ 2. Flow in a parallel plate channel with porous lining on one side

We consider the rectilinear flow of a viscous fluid through a two dimensional parallel channel formed by rigid impermeable walls at $y = 0$ and $y = h$ as shown in Fig. 1.

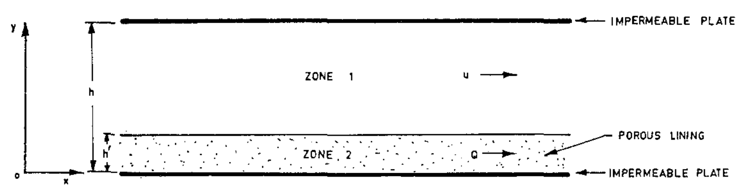

Fig. 1. Physical Model (Porous lining on one wall).

The lower wall is covered with a homogeneous and isotropic permeable material of thickness h' ($\neq 0$) thus dividing the flow region into two zones, Zone 1 denoting the region of the free flow between the upper impermeable wall and the nominal surface $y = h'$, and Zone 2 denoting the region of flow through the porous material.

2.1 *Mathematical formulation*

The flow which is caused by a uniform pressure gradient in the longitudinal direction in both the zones is assumed to be fully developed and the fluid properties are all assumed to be constant. Then the flow in Zone 1 is governed by the Navier-Stokes equation

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}, \tag{1}$$

and that in Zone 2 by the Darcy law

$$Q = -\frac{K}{\mu}\frac{dp}{dx}. \tag{2}$$

We solve (1) under the following boundary conditions:

$$u = 0 \quad \text{at} \quad y = h \tag{3}$$

and the BJ boundary condition

$$\frac{\mathrm{d}u}{\mathrm{d}y} = \frac{\alpha}{\sqrt{K}}(u_{\mathrm{B}} - Q) \quad \text{at} \quad y = h' \tag{4}$$

where the slip parameter α, a property of the porous material, is to be determined experimentally. Solution of (1) satisfying (3) and (4) is

$$v = (1 - \eta)\left[\frac{P}{12}(1 + \eta) - P\varepsilon + \frac{\alpha P}{\sigma} - \alpha\sigma v_{\mathrm{B}}\right] \tag{5}$$

where

$$v_{\mathrm{B}} = \frac{P(1 - \varepsilon)[\sigma(1 - \varepsilon) + 2\alpha]}{2\sigma[1 + \alpha\sigma(1 - \varepsilon)]} \tag{6}$$

and

$$(v, \eta, \xi, \pi, R, P, \sigma, Q', \varepsilon) =$$

$$= \left(\frac{u}{\bar{u}}, \frac{y}{h}, \frac{x}{h}, \frac{p}{\frac{1}{2}\rho\bar{u}^2}, \frac{\rho\bar{u}h}{\mu}, \frac{-R}{2}\frac{\partial\pi}{\partial\xi}, \frac{h}{\sqrt{K}}, \frac{Q}{\bar{u}}, \frac{h'}{h}\right). \tag{7}$$

It is important to note that the physical configuration is such that $0 < \varepsilon < 1$.

To find the quantitative effect of slip on the flow, we calculate the non-dimensional mass flow rate

$$M = M_1 + M_2 \tag{8}$$

where

$$M_1 = \int_{\varepsilon}^{1} v\,\mathrm{d}\eta = \frac{P}{12}(1 - \varepsilon)^3\left[\frac{4 + \alpha\sigma(1 - \varepsilon) - 6\alpha^2}{1 + \alpha\sigma(1 - \varepsilon)}\right] + \frac{P\alpha}{2\sigma}(1 - \varepsilon)^2 \equiv$$

$$\equiv \frac{P}{12}A + \frac{P}{2}B \tag{9}$$

and

$$M_2 = Q'\varepsilon = \frac{P\varepsilon}{\sigma^2}. \tag{10}$$

In order to bring out the effect of porous lining in the channel, we compare M with the mass flow rate M^* in the channel in the absence of lining where

$$M^* = \int_0^1 v \, d\eta = P/12 \tag{11}$$

and v is the non-dimensional solution of (1) under the boundary conditions $u = 0$ at $y = 0$ and at $y = h$. We note that α can be determined experimentally from the expression for M_1/M^* in which all the quantities are known except α.

In this paper since our main interest is to study the influence of ε on the mass flow rate, we compute M/M^* for the materials which are used in the experiments of Beavers and Joseph and for which the values of α are determined by them. The results are shown in Figs. 2 and 3.

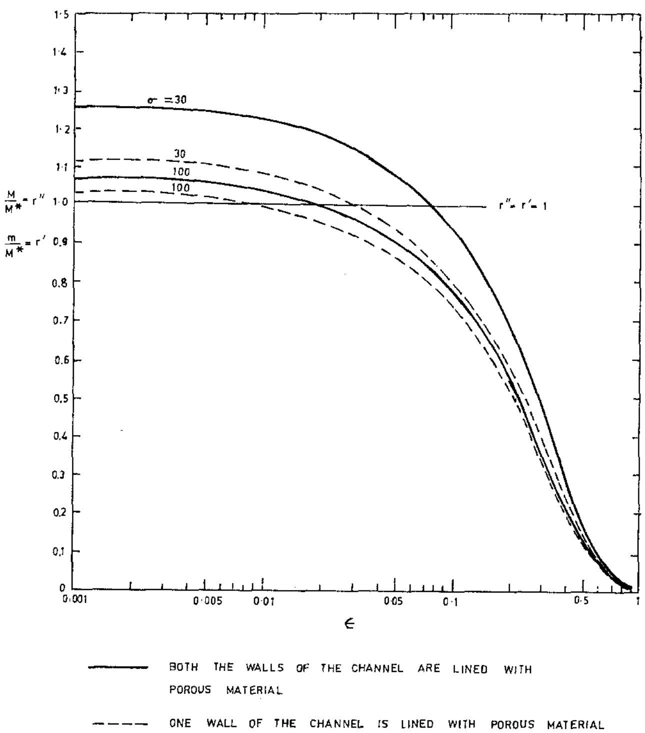

Fig. 2. Variation of m/M^*, M/M^* with ε for Foametal A: $K = 1.1 \times 10^{-5}$: $\alpha = 0.8$.

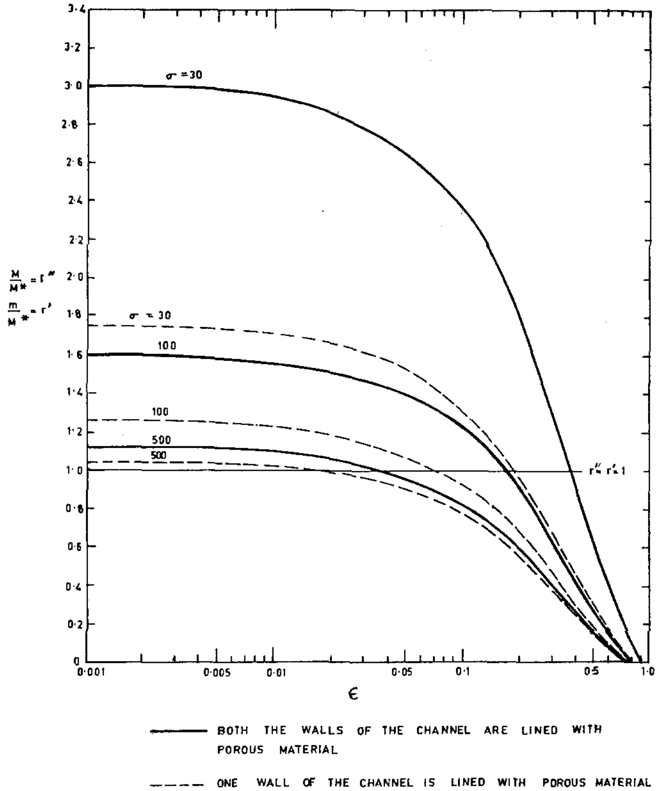

Fig. 3. Variation of m/M^*, M/M^* with ε for Aloxite: $K = 1.0 \times 10^{-6}$: $\alpha = 0.1$.

§ 3. Flow in a parallel plate channel with porous lining on both the sides

The flow situation considered here is the same as that discussed in § 2 except that both the bounding plates of the channel are covered by permeable material of thickness $h'/2$ as shown in Fig. 4.

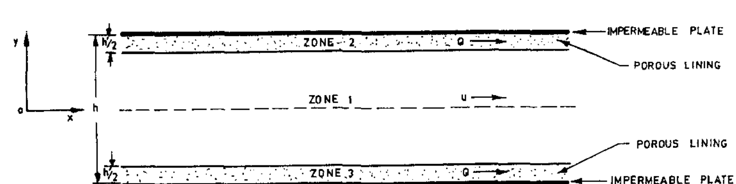

Fig. 4. Physical Model (Porous lining on both the walls).

In this case the solution of (1), using the boundary conditions

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{K}}(u_B - Q) \quad \text{at} \quad y = -\frac{h}{2} + \frac{h'}{2} \tag{12}$$

and

$$u = u_B \quad \text{at} \quad y = \frac{h}{2} - \frac{h'}{2} \tag{13}$$

is

$$v = -P\eta^2 + \frac{P}{2\sigma}[\sigma(\varepsilon - 1) - 2\alpha]\eta + $$
$$+ \frac{P}{8\sigma}[4\alpha + 3\sigma - (4\alpha + 6\sigma)\varepsilon + 3\sigma\varepsilon^2] + $$
$$+ \alpha\sigma v_B\left[\eta + \frac{1}{\alpha\sigma} + \frac{\varepsilon}{2} - \frac{1}{2}\right] \tag{14}$$

where

$$v_B = \frac{P}{2\alpha\sigma^2}[\sigma(1 - \varepsilon) + 2\alpha], \qquad 0 < \varepsilon < 1 \tag{15}$$

and the non-dimensional quantities are as defined in (7).

To know the effect of porous lining on both sides of the channel we can compare, as before, the total non-dimensional mass flow rate m in the channel, with M^* where

$$m = m_1 + m_2 + m_3 \tag{16}$$

$$m_1 = \int_{-\frac{1}{2}+(\varepsilon/2)}^{\frac{1}{2}-(\varepsilon/2)} v\, d\eta = \frac{P}{12}\left[(1-\varepsilon)^3 + \frac{6}{\alpha\sigma}(1-\varepsilon)^2 + \frac{12}{\sigma^2}(1-\varepsilon)\right] \equiv$$
$$\equiv \frac{P}{12}C \tag{17}$$

$$m_2 = m_3 = \frac{P\varepsilon}{2\sigma^2}. \tag{18}$$

Here again we compute m/M^* for materials which are used in the experiments of Beavers and Joseph, for which the values of α are known. The results are shown in Figs. 2 and 3.

§ 4. Resistance coefficients

The above analysis is based on the assumption of laminar flow. Therefore, for practical purpose it is important to know the conditions under which the laminar flow breaks down. For this purpose, we calculate the Chezy resistance coefficient λ defined by the non-dimensional pressure gradient [10]

$$\lambda = -\frac{\partial \pi}{\partial \xi} = \frac{2P}{R} \tag{19}$$

where π and ξ are given by (7).

To calculate the product λR for different cases, we make use of the following relations

$$\int_{\varepsilon}^{1} v \, \mathrm{d}\eta = 1 - \varepsilon, \tag{20}$$

$$\int_{-\frac{1}{2}+(\varepsilon/2)}^{\frac{1}{2}-(\varepsilon/2)} v \, \mathrm{d}\eta = 1 - \varepsilon, \tag{21}$$

and

$$\int_{0}^{1} v \, \mathrm{d}\eta = 1, \tag{22}$$

which hold good respectively for flows:

(i) in Zone 1 of Fig. 1, (ii) in Zone 1 of Fig. 4 and (iii) in the channel of height h without any lining.

The above relations follow from the definition of average velocity and the non-dimensionalisation (7). We thus get from (9), (11), (17) and (19)–(22),

$$(\lambda R)_1 = \frac{24(1 - \varepsilon)}{A + 6B} \tag{23}$$

$$(\lambda R)_2 = \frac{24(1 - \varepsilon)}{C} \tag{24}$$

$$(\lambda R)_* = 24. \tag{25}$$

Thus,

$$\frac{(\lambda R)_1}{(\lambda R)_*} = \frac{1 - \varepsilon}{A + 6B} = \frac{1 - \varepsilon}{(M_1/M^*)} \tag{26}$$

and

$$\frac{(\lambda R)_2}{(\lambda R)_*} = \frac{1 - \varepsilon}{C} = \frac{1 - \varepsilon}{(m_1/M^*)}. \tag{27}$$

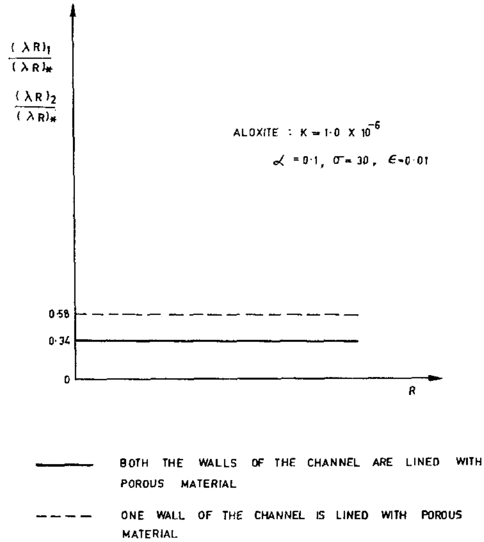

Fig. 5. Comparison of Resistance Coefficients in flows involving one and two-side porous lining.

The ratios

$$\frac{(\lambda R)_1}{(\lambda R)_*} \quad \text{and} \quad \frac{(\lambda R)_2}{(\lambda R)_*}$$

are numerically evaluated for different values of α, σ and ε. Typical behaviour of these coefficients is shown in Fig. 5.

§ 5. Discussion of results

In this section, we compare the advantages of lining the channel on one side vis a vis the lining on both the sides. In view of the fact that the mass flow rates M_2, m_2 and m_3 through the porous linings are very small compared to the mass flow rates M_1 and m_1 in the free zones, we compare the mass flow rates M_1 and m_1 and obtain

$$\frac{m_1}{M_1} = 1 + \frac{3\alpha\sigma^2(1-\varepsilon)^2 + 6\alpha^2\sigma(1-\varepsilon) + 6\sigma(1-\varepsilon) + 12\alpha}{4\alpha\sigma^2(1-\varepsilon)^2 + \alpha^2\sigma^3(1-\varepsilon)^3 + 6\alpha^2\sigma(1-\varepsilon)}, \quad 0 < \varepsilon < 1. \tag{28}$$

Since the second term on the right hand side of (28) is always positive, we conclude that m_1 is always greater than M_1. This means that using the same quantity of lining material we can achieve greater mass flow rate by lining both walls of the channel (§ 3) as compared to the case where only one wall of the channel is lined (§ 2).

Numerical values of the mass flow rates have been obtained for different combinations of the parameters α, σ and ε and the results are shown in Figs. 2 and 3. From these it is clear that, though the thickness of the permeable material contributes to the reduction of the region of free flow, the effect of the slip will be such that for a given material the mass flow rate with the porous lining will be greater than that without porous lining up to a certain value of ε.

We also observe from these figures that for a fixed ε, the mass flow rate decreases with increasing σ and for a given σ, the mass flow rate decreases with increasing α.

From Figs. 2 and 3 we also conclude that the mass flow rate is greater in the case of the lining of both the walls than that in the case of the lining of only one wall.

Finally, from Fig. 5, we conclude that the breakdown of laminar flow occurs earlier in the case of lining both the walls with porous material than in the case of lining only one wall. However, the critical Reynolds number at which the assumption of laminar flow breaks down can be determined experimentally or through stability analysis.

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