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Performance of Iteratively Decoded Parallel Concatenated Error Control Codes in Phase Noise Corrupted BPSK Systems

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Iterative or Turbo decoding of parallel concatenated binary error control codes in Binary Phase Shift Keyed (BPSK) Communication System corrupted by both Additive White Gaussian Noise (AWGN) and oscillator phase noise is discussed. As an example of application of the discussed theory, iterative decoding technique of Parallel Concatenated block Codes (PCBC) is applied to a coherent Optical Code Division Multiple Access (OCDMA) network in the presence of Multiple Access Interference (MAI), detector shot noise and laser phase noise. Remarkable improvement in performance resulting from the use of turbo decoding in the OCDMA network is also discussed.

Indexing terms: Iterative (turbo) decoding, Log likelihood ratio, Phase noise, PDF, Coherent OCDMA.

1 INTRODUCTION

ITERATIVE or turbo decoding of parallel concatenated error control codes, popularly known as Turbo Codes [1-3], have generated a lot of interest in the coding community with their astonishing performance of a Bit Error Rate (BER) of 10^{-5} being achievable at a bit energy to noise density ratio of the order of 0.7 dB [4]. Though a lot of papers on turbo codes have appeared in the near past, almost all of them deal with situations where the transmissions are corrupted by only additive noise sources of a well defined probability density function (pdf). In practice however, in a coherent/synchronous communication system, the transmissions may as well be contaminated by random processes such as oscillator phase noise which gets multiplied with the transmitted data bits.

In the present work, the authors have taken this aspect into account and derived an explicit expression for the pdf of the correlation detector soft output in a coherent Binary Phase Shift Keyed (BPSK) receiver in the presence of both Additive White Gaussian Noise (AWGN) and oscillator phase noise. An exact knowledge of this pdf of the soft output of the detector is a must for an efficient and successful decoding of turbo codes.

As an example of the developed theory, iterative decoding of Parallel Concatenated Block codes (PCBC) is employed in a coherent Optical Code Division Multiple Access (OCDMA) network in the presence of Multiple Access Interference (MAI), detector shot noise and laser phase noise.

COMPUTATION OF LIKELIHOOD RATIO

A detailed discussion of turbo codes or iterative decoding technique is not attempted here since it can be easily found in the existing literatures [1-4]. Suffice it to say from the present discussion's viewpoint that iterative decoding invariably involves the computation of a Log Likelihood Radio (LLR) defined as

$$L_c(x_k) = \log_e \left[\frac{Px_k(x_k/d_k=1)}{Px_k(x_k/d_k=-1)} \right] \tag{1}$$

where, d_k is the kth binary data, x_k is the corresponding contaminated soft output from the detector and $Px_k(x_k/d_k=i)$ is the conditional pdf of x_k given that $d_k=i$. It is explicit from (1) that an exact knowledge of the pdf of the detector output is a must in order to compute the LLR $L_c(x_k)$.

In the following, the authors discuss the computation of

$L_c(x_k)$ in a coherent BPSK communication system corrupted by both AWGN and oscillator phase noise.

From elementary theory of data reception in coherent/synchronous communications as documented in the digital communications literature [5], it can be shown that the output x_k of a correlation detector in a BPSK receiver is given by

$$x_k = d_k \cos\phi + N \qquad (2)$$

where, d_k is as explained earlier, ϕ is the phase error between the received carrier and the local oscillator (assumed to remain constant within a bit interval), and N is a Gaussian random variable (rv) characterizing the AWGN contribution with zero mean and variance σ_N^2.

Under such ideal circumstances where the local oscillator phase is made to track the received carrier phase perfectly (with $\phi = 0$ or a constant very near to zero), the sole contaminating source will be the AWGN and the *LLR* $L_c(x_k)$ can be readily computed. However, when there are random fluctuations in ϕ, the pdf of x_k, which will be the convolution of the pdf of the first and second terms in the RHS of (2), has to be derived before computing $L_c(x_k)$.

The pdf of the rv $Y = \cos\phi$, as explained earlier, can be derived as given in the appendix and is found to be

$$p_y(y) = \frac{2}{\sqrt{1-y^2}} p_\Phi(\cos^{-1} y) \qquad -1 < y < +1 \qquad (3)$$

where, $p_\Phi(\cos^{-1} y)$ is the pdf expression of ϕ with the independent variable replaced by $\cos^{-1} y$.

The pdf of $Y + N$ will now be given by

$$P_{Y+N}(y+n) = \frac{2}{\sqrt{1-y^2}} p_\Phi(\cos^{-1} y) * P_N(n) \qquad (4)$$

where, $p_N(n)$ is the pdf of N and $*$ is the convolution operator.

As per the central limit theorem [6], since N is a Gaussian rv, the pdf of x_k will be comprising of two Gaussian functions centered at the mean of the rv $+\cos\phi$ and $-\cos\phi$ respectively with their individual variances equal to one half the sum of the variance of $+\cos\phi$ and variance of N and one half the sum of the variance of $-\cos\phi$ and the variance of N respectively.

As an example, assuming that ϕ has a Gaussian pdf given by

$$p_\Phi(\phi) = \frac{1}{\sqrt{2\pi\sigma_\phi^2}} \exp\left(\frac{-\phi^2}{2\sigma_\Phi^2}\right) \qquad (5)$$

the pdf of $Y = \cos\phi$ will be

$$p_Y(y) = \sqrt{\frac{2}{\pi(1-y^2)\sigma_\phi^2}} \exp\left(\frac{-\cos^{-1}(y)^2}{2\sigma_\phi^2}\right) \quad -1 < y < +1 \qquad (6)$$

A plot of (6) with $\sigma_\phi^2 = 1$ is as shown in Fig 1.

As mentioned earlier, by virtue of the central limit theorem, the pdf of x_k will be Gaussian. However, to get the exact expression for the pdf of x_k, the mean and variance of (3) has to be known, the computation of which may not be possible analytically.

To solve this problem, for the specific case where ϕ has a Gaussian pdf as given in (5), the mean and variance of $Y = \cos\phi$ for different values of the phase error standard deviation σ_ϕ was computed by simulation and plotted as a function of σ_ϕ. Based on the shape of these graphs, explicit expressions for the mean and variance of $Y = \cos\phi$ as functions of σ_ϕ were assumed. To check the validity of these assumptions, the assumed expressions for mean and

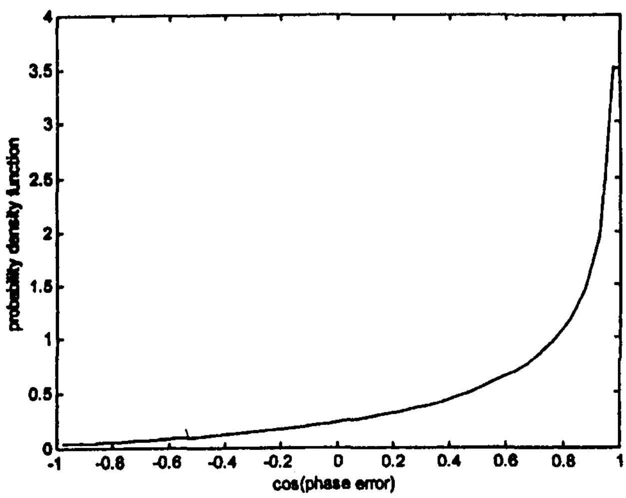

Fig 1 Probability density function of cos (ϕ) with a zero mean and unit variance Gaussian distribution for the phase error ϕ

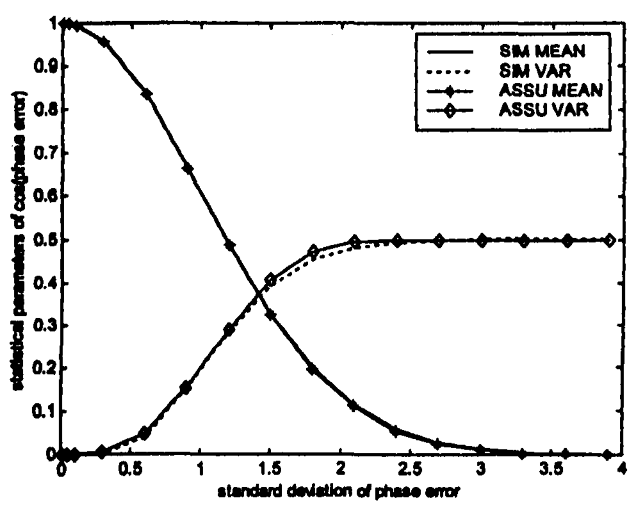

Fig 2 Simulated mean and Simulated variance of cos (ϕ) along with the assumed mean and assumed variance of cos (ϕ) as a function of the standard deviation σ_ϕ of the zero mean Gaussian distributed phase error ϕ

variance were plotted along with the simulated values. The results are reported in Fig 2 and a perfect agreement between the simulated results and the assumed expressions are seen to exist. The assumed mean $\mu_Y(\sigma_\phi)$ and variance $\sigma_Y^2(\sigma_\phi)$ of $Y = \cos\phi$ which were plotted in Fig 2 along with the simulation results were,

$$\mu_Y(\sigma_\phi) = \exp\left(\frac{-\sigma_\phi^2}{2}\right) \tag{7}$$

and

$$\sigma_Y^2(\sigma_\phi) = \left(\frac{1-\exp(-\sigma_\phi^3/2)}{2}\right) \tag{8}$$

respectively.

From these discussions, it can now be shown that the conditional pdf of x_k is given by,

$$p_{x_k}(x_k / d_k = +1) = \frac{1}{\sqrt{2\pi(\sigma_Y^2(\sigma_\phi) + \sigma_N^2)}} \exp\left(\frac{-(x_k - \mu_Y(\sigma_\phi))^2}{2(\sigma_y^2(\sigma_\phi) + \sigma_N^2)}\right) \tag{9}$$

and

$$p_{x_k}(x_k / d_k = -1) = \frac{1}{\sqrt{2\pi(\sigma_Y^2(\sigma_\phi) + \sigma_N^2)}} \exp\left(\frac{-(x_k + \mu_Y(\sigma_\phi))^2}{2(\sigma_y^2(\sigma_\phi) + \sigma_N^2)}\right) \tag{10}$$

and the LLR $L_c(x_k)$ is given by

$$L_c(x_k) = \frac{2x_k\mu_Y(\sigma_\phi)}{(\sigma_Y^2(\sigma_\phi) + \sigma_N^2)} \tag{11}$$

TURBO CODED COHERENT OCDMA NETWORK

As an example of the applica;tion of the theory discussed in the previous section, some results of applying turbo coding to a coherent OCDMA network is discussed in this section.

The scheme of the employed encoding technique is as outlined in Fig 3. The data bits stored in a 4 × 4 array and identified as d_{jk} are first encoded vertically, column wise, by a (7,4) BCH code [7] and after interleaving as given in Fig 3, encoded again horizontally, row wise, using the same (7,4) BCH code. Thus for vertical encoding the data bits are picked up from the 4 × 4 array as

$$d_{0i}\ d_{1i}\ d_{2i}\ d_{3i} \quad \text{for } i = 0,1,2,3$$

and for horizontal encoding, the data bits are picked up from the 4 × 4 array as

$$d_{(i\,mod\,4)\,0}\ d_{((i+1)\,mod\,4)\,1}\ d_{((i+2)\,mod\,4)\,2}\ d_{((i+3)\,mod\,4)\,3} \quad \text{for } i = 0,1,2,3$$

The interleaving of data bits as explained above has the ability to correct rectangular error patterns [8] which may not be correctable if a simple interleaving technique wherein the data bits for the horizontal encoding were read from the array in the same fashion as it was done in the case of vertical encoding.

Thus with a (7,4) BCH code generator matrix G as given below

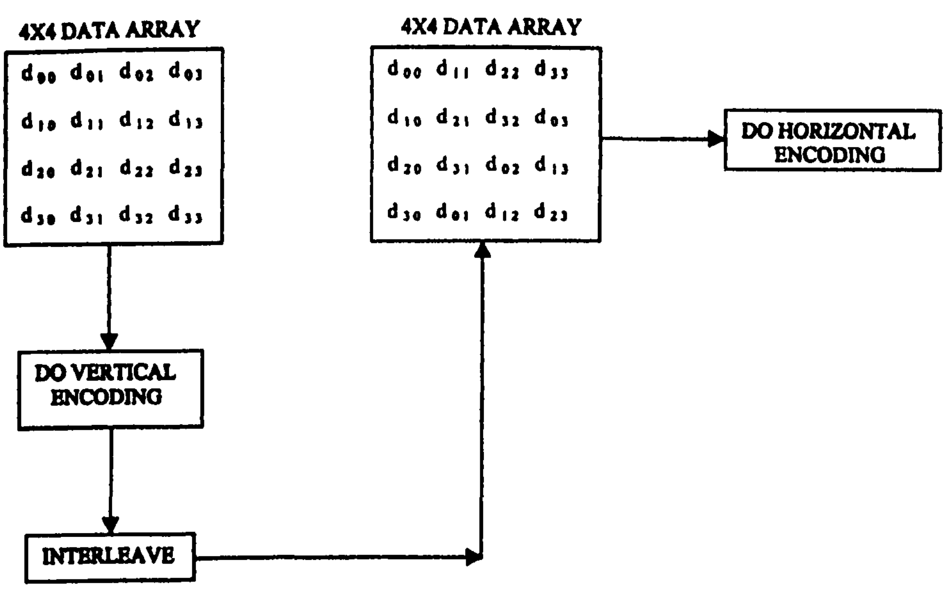

Fig 3 Parallel concatenated error control coding scheme with 4×4 data array and (7,4) BCH code

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{12}$$

the three vertically encoded parity bits for the ith column will be,

$$\begin{aligned} v_{0i} &= d_{0i} \oplus d_{1i} \oplus d_{2i} \\ v_{1i} &= d_{1i} \oplus d_{2i} \oplus d_{3i} \\ v_{2i} &= d_{0i} \oplus d_{1i} \oplus d_{3i} \end{aligned} \tag{13}$$

and the three horizontally encoded parity bits for the ith row will be,

$$\begin{aligned} h_{i0} &= d_{(i\,mod\,4)\,0} \oplus d_{((i+1)\,mod\,4)\,1} \oplus d_{((i+2)\,mod\,4)\,2} \\ h_{i1} &= d_{((i+1)\,\mathrm{mod}\,4)\,1} \oplus d_{((i+2)\,\mathrm{mod}\,4)\,2} \oplus d_{((i+3)\,\mathrm{mod}\,4)\,3} \\ h_{i2} &= d_{(i\,\mathrm{mod}\,4)\,0} \oplus d_{((i+1)\,\mathrm{mod}\,4)\,1} \oplus d_{((i+3)\,\mathrm{mod}\,4)\,3} \end{aligned} \tag{14}$$

In a coherent OCDMA network employing a Decision Directed Phase Locked Loop (DDPLL) [9-11] for optical carrier phase estimation, it can be shown that [12] the correlation detector output of the homodyne receiver 'tuned' to the rth user out of a total K users is given by

$$I = d_r \cos\phi_r + \frac{1}{T}\int_0^T \left(\sum_{i=1, i\neq r}^{K} S_r(t)\, C_i(t-\tau_i)\cos\phi_i \right) dt + \frac{1}{2.R.\sqrt{P_s P_L T}} \int_0^T n(t)\, dt \tag{15}$$

where

$d_r =$ Bipolar data bit of interest of the rth user

$\phi_j =$ Phase error between the received optical carrier of jth user and local laser

$T =$ Bit duration (signaling interval)

$S_r(t) =$ Time domain bipolar representation of rth user's spreading code with a chip duration of T_c

$Ci(t) = Si(t).di(t)$ with $di(t)$ being the time domain bipolar representation of the ith user's data sequence with a bit duration of T

$\tau_i =$ Time delay, distributed uniformly between 0 to T of the ith user's signal

$R =$ Common responsivity of all the photodetectors used in the receiver

P_s and $P_L =$ Power received per user and local laser power respectively

$n(t) =$ Sample function of detector shot noise process.

The variance σ_M^2 of the second term in the RHS of (15) which accounts for the MAI and the variance σ_{SN}^2 of third term in the RHS of (15) which accounts for the detector shot noise, both of which are additive interferences, have been shown to be [10-13].

$$\sigma_M^2 = \frac{K-1}{3.N} \tag{16}$$

with $N = T/T_c$

and

$$\sigma_{SN}^2 = \frac{1}{4.\,(\text{photons / bit})} \tag{17}$$

The phase error ϕ_r has a Viterbi-Tikhonov distribution [14] with a pdf as given by [10-12]

$$p_\Phi(\phi_r) = \frac{1}{2\pi I_o(\alpha)} \exp(\alpha \cos\phi_r) \qquad -\pi < \phi_r < +\pi \tag{18}$$

where, α is the maximum loop Signal to Noise Ratio (SNR)

$$\alpha = \frac{1}{\sqrt{\pi.\,\Delta f.T\left(\frac{1}{2.\,\text{photons /bit}} + \frac{2.(K-1)}{3.N}\right)}} \tag{19}$$

with Δf being the sum of the 3dB linewidths of the transmitting and local laser and $I_o()$ being the modified Bessel function of the zeroth order.

For comparatively large values of α , (18) can be approximated as a Gaussian pdf with zero mean and variance equal to $1/\alpha$ [10]. Since, for reasonably large number of users, the MAI has a Gaussian pdf [13-15], the additive interferences in the OCDMA network under consideration here will have a Gaussian pdf and as a result, the theory discussed in the last section and the results arrived at thereby, are applicable to the present system with σ_ϕ^2 and σ_N^2 being replaced by $(1/\alpha)$ and $(\sigma_M^2 + \sigma_{SN}^2)$ respectively.

Thus the equivalent LLR $L_c(I)$ of $L_c(x_k)$ as discussed in the last section for the present OCDMA network will be given by

$$L_c(I) = \frac{2.I.\mu_Y(\sqrt{1/\alpha})}{\sigma_Y^2(\sqrt{1/\alpha}) + \sigma_{SN}^2 + \sigma_M^2} \tag{20}$$

The numerical results pertaining to the performance of this turbo encoded OCDMA network is discussed in the next section.

RESULTS

Figure 4 shows the BER as a function of Δf with $K = 129$, $N = 127$, bit rate = 150 Mega bits per second (Mbps) and 1500 photons/data bit being detected. It can be seen that the turbo coded network performance is much more superior to the uncoded network performance. For comparison, the figure also reports the BER performance of an uncoded OCDMA network, which occupies the same bandwidth as that occupied by the turbo coded OCDMA network, by virtue of an increase in N by a factor $1/R_c$, with R_c being the code rate, which is 0.4 in the present case. It may be noted that the turbo coded OCDMA network performance is still much superior to the uncoded OCDMA network. Figure 5 reports similar results with $K = 100$.

Figure 6 and Fig 7 report the BER performance of the turbo coded and uncoded OCDMA network as a function of K for $\Delta f = 20$ MHz and 16 MHz respectively. A similar comparison as to earlier of the turbo encoded OCDMA network performance with an uncoded OCDMA network with $N = 127 \times 0.4$ is also given in the figure.

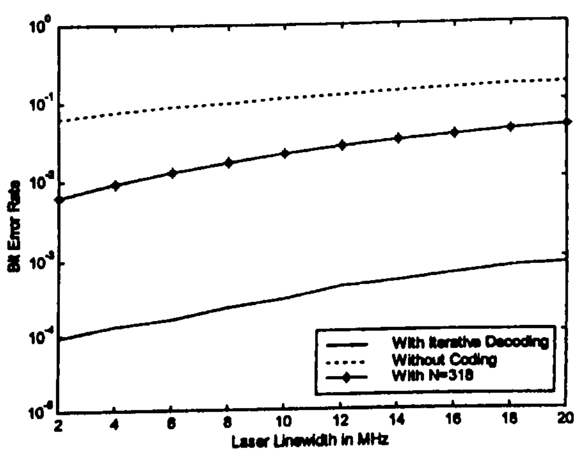

Fig 4 Coherent OCDMA network BER performance with and without error control coding as a function of laser linewidth with number of users =129 and photons per information bit fixed as 1500

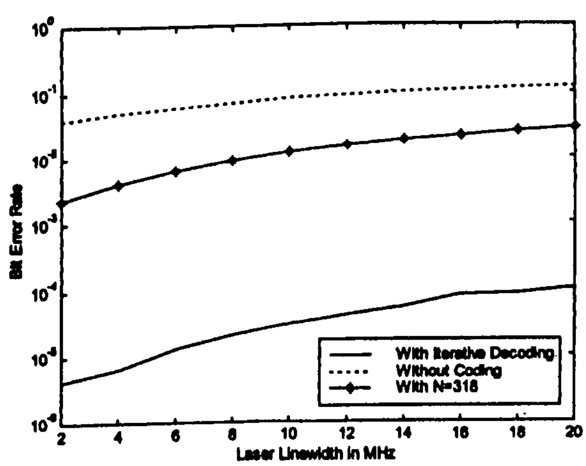

Fig 5 Coherent OCDMA network BER performance with and without error control coding as a function of laser linewidth with number of users=100 and photons per information bit fixed as 1500

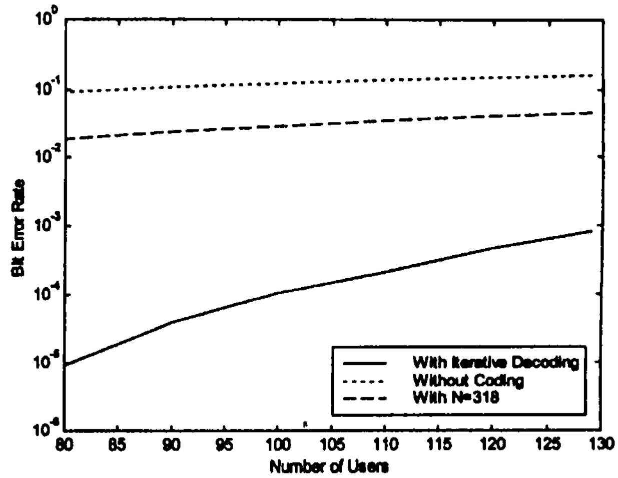

Fig 6 Coherent OCDMA network BER performance with and without error control coding as a function of number of users with laser linewidth =20 MHz and photons per information bit fixed as 1500

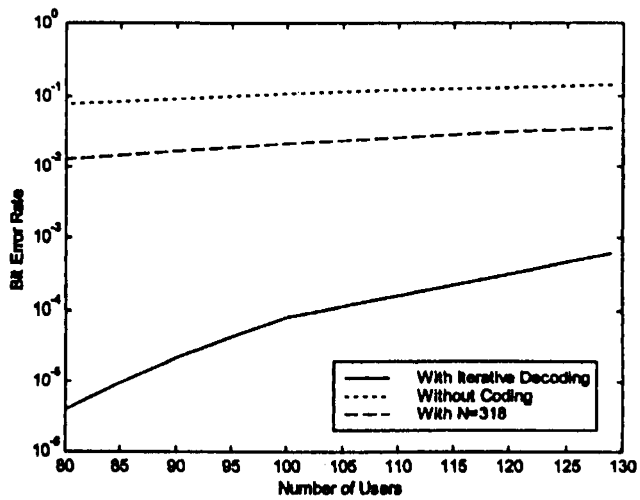

Fig 7 Coherent OCDMA network BER performance with and without error control coding as a function of number of users with laser linewidth =16 MHz and photons per information bit fixed as 1500

In the results discussed under this section, the authors have deliberately considered the scenario where the bit energy to noise density ratio and loop SNR of the DDPLL are relatively poor. This was mainly due to two reasons, the first being that this helps to bring out the superiority of iterative decoding techniques in a more convincing manner and the second reason being the ease in carrying out the simulations.

CONCLUSION

The problem of iterative decoding of parallel concatenated error control codes in the presence of both AWGN and oscillator phase noise was discussed. More specifically, the computation of the LLR pertaining to the correlation detector soft output was discussed. As an example of the application of the discussed theory, iterative decoding of PCBC in a coherent OCDMA network was discussed. Several comparison between the coded and uncoded OCDMA network performance were also presented.

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Appendix

In this appendix, the derivation of the pdf $p_Y(y)$ of the rv $Y = \cos\phi$ is discussed. From the definition of pdf, we have,

$$P_Y(y) = \mathrm{Lt}_{\Delta y \to 0} \frac{1}{\Delta y} P\,(y < Y < y + \Delta y)$$

$$= \mathrm{Lt}_{\Delta y \to 0} \frac{1}{\Delta y} [p_\Phi(\theta < \phi < (\theta + \Delta\theta)) + p_\Phi(-(\theta+\Delta\theta) < \phi, < -\theta)] \tag{A1}$$

where, θ refers to a particular value taken by the rv ϕ .

Equation (A1) can be further expressed as

$$p_Y(y) = \mathrm{Lt}_{\Delta y \to 0} \frac{1}{\Delta y} \left[\int_0^{\theta+\Delta\theta} p_\Phi(\phi)\, d\phi + \int_{-\theta-\Delta\theta}^{-\theta} p_\Phi(\phi)\, d\phi \right] \tag{A2}$$

If $p_\Phi(\phi)$ is an even and symmetric function,

$$p_Y(y) = \mathrm{Lt}_{\Delta y \to 0} \frac{2}{\Delta y} [p_\Phi(\theta + \Delta\theta) - p_\Phi(\theta)] \tag{A3}$$

where, $P_\Phi(\theta)$ is the distribution function defined as

$$P_\Phi(\theta) = \int_{-\infty}^{\theta} P_\Phi(\phi)\, d\phi \tag{A4}$$

Further,. since $Y = \cos\phi$,

$$p_Y(y) = \mathrm{Lt}_{\Delta y \to 0} \frac{2}{\Delta y} [P_\Phi(\cos^{-1} y) - p_\Phi(\cos^{-1}(y + \Delta y))]$$

$$= 2 \frac{d}{dy} p_\Phi(\cos^{-1} y) \tag{A5}$$

Carrying out the differentiation as given in (A5), we get,

$$p_Y(y) = \frac{2}{\sqrt{1 - y^2}} p_\Phi(\cos^{-1} y) \tag{A6}$$

which is the desired result.