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OPTIMUM DESIGN OF TROUGH TYPE FOLDED PLATE ROOFS

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Abstract—Optimization of simply-supported symmetrical trough type folded plate roofs using improved move-limit method of sequential linear programming and sequential unconstrained minimization technique is discussed. Improved move-limit method of sequential linear programming has been found to be suitable for optimization of trough type folded plate roofs and using the same, the effect of cost ratio on optimum design variables and the effect of the number of trough units for a given span on optimum design have been studied and discussed. Optimum dimensions have been prepared for various spans normally encountered.

INTRODUCTION

Folded plates are a very useful form of structure for roofing large column-free area. Among the different types, trough type folded plates are commonly used for roofing such large spans. The analysis and design of this roof consists in assuming a preliminary section and checking the stresses. Since the analysis is a difficult task without a computer, the designer will have a tendency to assume an oversafe preliminary section and satisfy himself that the design is safe. When a computer is made use of for the design, one can go in for producing optimum designs by linking the analysis and design software with a suitable optimization technique. But the choice of a technique depends on the type of problem and it is difficult to categorically specify a particular solution technique for all cases. In this paper the capability of the two optimization techniques viz. (i) the improved move limit method of sequential linear programming (SLP) and (ii) sequential unconstrained minimization technique (SUMT) for the optimum design of trough type folded plates is investigated and discussed. Simpson's method has been used for the analysis and the design is done in accordance with IS 2210 [1].

PROBLEM FORMULATION

A trough type folded plate is defined by a set of parameters such as the span, width, the number of plates, width of individual plates, thickness of the plates and height. Formulation of the optimum design problem consists in identifying the design variables, constraints and the objective function.

DESIGN VARIABLES

The cross section of a typical trough type folded plate roof is shown in Fig. 1. The various parameters to be selected are the width of individual plates, angle of inclination of the inclined plates and the thickness of the plates. From aesthetic consideration, the width of all top plates are kept the same (b_1). Similarly, width of all bottom plates is kept as b_2 and inclination of all inclined plates is selected as θ to the horizontal (Fig. 1). The width of the last bottom plate is kept as half the width of the inner bottom plates. Thickness of plates is maintained uniformly throughout. Since the inclination of all plates are the same, the horizontal projection and width will be the same for all inclined plates. Hence for a given width of the roof, if b_1, horizontal projection and number of plates are known, b_2 can be determined and it need not be considered as an independent variable. Thus the design variables selected for this type of roof are:

(i) width by top plates b_1 (X_1) in mm;
(ii) horizontal projection (X_2) in mm;
(iii) thickness of plates (X_3) in mm;
(iv) angle γ_1 (X_4) in degrees.

Hence a design vector is defined as (X_1, X_2, X_3, X_4) and these are shown in Fig. 2.

CONSTRAINTS

The compressive stress at any point should be less than the permissible compressive stress in concrete. Since the stresses are maximum at midspan, the

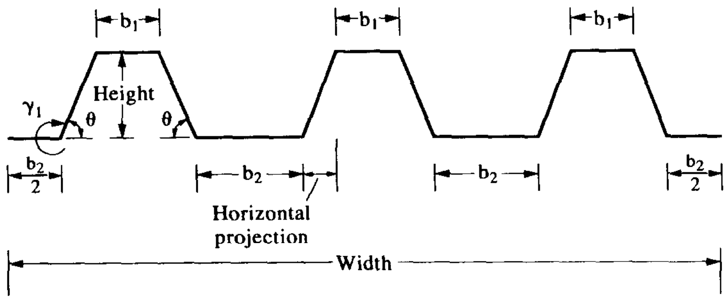

Fig. 1. Section of a trough type folded plate roof.

maximum compressive stress at midspan (σ) is computed and the constraint is imposed as

$$\sigma - \sigma_{cb} \leqslant 0 \qquad (1)$$

where σ_{cb} = permissible compressive stress in concrete.

The maximum width of an individual plate is kept as Span/3 in order to eliminate two-way behavior and deep beam action of the individual slabs. A minimum thickness of 75 mm for the plates has been kept as per IS 2210 [1]. In order to avoid backforms while concreting, an upper limit of 40° for angle θ (Fig. 1) has been imposed.

OBJECTIVE FUNCTION

The material cost is taken as the objective function to be minimized.

The material cost can be written as

$$C = (V_c + V_s R)C_c \qquad (2)$$

where C = total material cost; V_c = volume of concrete; V_s = volume of steel; R = cost ratio (cost of steel per unit volume/cost of concrete per unit volume); C_c = cost of concrete per unit volume.

Since C_c is constant for a given problem, minimizing C is equivalent to minimizing

$$F = V_c + V_s R \qquad (3)$$

where F is taken as the objective function to be minimized.

OPTIMIZATION TECHNIQUES

A general constrained optimization problem may be stated as

$$\text{find } \mathbf{X} = [x_1 x_2 \ldots x_n] \text{ which minimizes } F(X)$$

subject to the constraints $G_j(X) \leqslant 0$, $j = 1, 2, \ldots m$ where $\mathbf{X}$ is an n-dimensional vector called the design vector, $F(X)$ is the objective function and G_js are the constraints.

IMPROVED MOVE LIMIT METHOD OF SLP

The sequential linear programming consists in linearizing the objective function and the constraints in the vicinity of a design vector and solving the resulting linear programming problem to get a new design vector. The sequence of linearizing and solving the linear programming problem is continued from the new point till optimum is reached.

If the optimum does not lie at the corner of the constraint surfaces, the solution starts oscillating between two points. Griffith and Stewart [2] suggested the move-limit method to avoid oscillation of the solution between two points. Since the linearization of the constraints holds good only in the neighbourhood of the design point, additional constraints called move limits have been imposed in the resulting linear programming problem to restrict the movement of the design vector. These additional constraints for the Kth, iteration may be written as

$$x_i^{K+1} - x_i^K \leqslant M_i^K \qquad (4)$$

where x_i^{K+1}, x_i^K and M_i^K are the ith components of $\mathbf{X}^{K+1}$, $\mathbf{X}^K$ and $\mathbf{M}^K$. The vector $\mathbf{M}^K$ prescribes the move-limits on the design variables.

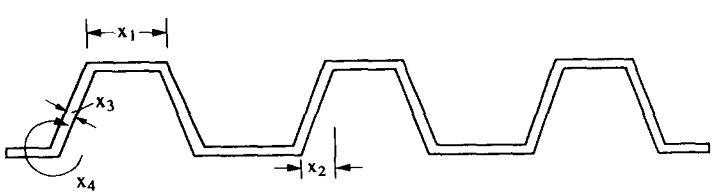

Fig. 2. Design variables for symmetrical trough type folded plate roof.

This move-limit method has been used by Pope [3, 4] for structural optimization. The suggestions given by Pope for the different situations encountered while using this algorithm for minimization are as follows.

In the Kth iteration, if the design vector obtained after linear programming solution, i.e. $\mathbf{X}^{K+1}$, is found to be feasible, and shows no improvement in objective function, i.e. $F(\mathbf{X}^{K+1}) > F(\mathbf{X}^{K})$, Pope has suggested the interval and move-limit halving technique to get a new design point, i.e. to take the design vector as $0.5\ (X^{K} + \mathbf{X}^{K+1})$ and halve the move-limits. Still if no improvement is found, the interval and move-limit halving techniques are to be continued till improvement is found.

If $\mathbf{X}^{K+1}$ is not feasible, Pope has suggested moving along the line joining the origin and the point to steer the point to the feasible region till just feasible point is reached.

Ramakrishnan and Bhavikatti [5, 6] while studying this algorithm noticed that certain improvements could be made which could result in faster convergence to optimum. They have improved the conventional move-limit method of SLP (as used by Pope) and this improved version is called the improved move-limit method of sequential linear programming in the literature. The improvements suggested by them in the move-limit method to make it more efficient are:

(i) The use of quadratic interpolation instead of interval and move-limit halving technique when a linear programming solution gives a feasible design point with a higher value of the objective function.

(ii) Moving along the gradient direction of the most violated constraint at the previous design point instead of moving in the direction joining the origin and the point for steering an infeasible design point to the feasible region.

(iii) Checking the usability of the new direction before going for quadratic interpolation if it is found that the objective function is higher at a design point obtained after steering to the feasible region.

A detailed description of this method is available in [6].

SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE (SUMT)

In this technique, a composite function is constructed using the objective and constraint functions which also contain a parameter called the penalty parameter. Once the composite function is defined, it is minimized using any of the unconstrained minimization techniques. If the unconstrained minimization of the composite function is repeated for a sequence of values of the penalty parameter, the solution may be brought to converge to that of the original problem.

SUMT consists of two basically different types of penalty functions, based on which the methods are called penalty function method (exterior methods) or barrier function method. For the present work, the SUMT algorithm due to Fiacco and McCormick as given by Kuester and Mize [7] has been used.

This is the interior (barrier) function method for problems with mixed equality and inequality constraints. The program finds the minimum of a multivariable, non-linear function subject to non linear inequality and equality constraints given by

$$\begin{aligned} &\text{Minimize } F(x_1, x_2, \ldots x_n) \\ &\text{subject to } G_K(x_1, x_2, \ldots x_n) \geqslant 0,\ K = 1, 2, \ldots M \text{ and} \\ &\quad H_K(x_1, x_2, \ldots x_n) = 0, K = M+1, \\ &M+2 \ldots M+P \ldots \end{aligned} \tag{5}$$

A modified objective function (composite function) is formulated consisting of the original function and penalty functions with the form

$$P = F - r \sum_{K=1}^{M} l_n \quad G_{K+} \sum_{K=M+1}^{M+P} H_K^2/r \tag{6}$$

where P = modified objective function; F = original objective function; r = a positive constant known as penalty parameter.

As the algorithm progresses, r is reevaluated to form a monotonically decreasing sequence $r_1 > r_2 > \cdots > 0$. As r becomes small, under suitable conditions P approaches F and the problem is solved. The unconstrained minimization is carried out by using McCormick's modification of the Fletcher–Powell method with golden section method of one dimensional search.

OPTIMUM DESIGN PROBLEM

The progress of optimization using the two techniques i.e. improved move-limit method of SLP and SUMT and convergence to global optimum, is illustrated using a numerical example. The program is run on a mini-computer with two processing units with microprocessors MC 68030 and Coprocessors MC 68882. The details of the problems are:

Span = 18 m,
Width = 6.4 m,
Number of plates = 9 (two trough units),
X_1 = 480 mm,
X_2 = 1110 mm,
X_3 = 100 mm,
$X_4 = 320^{\circ}$,
Live load = 0.7 kN m^{-2}.

Thus the initial design point is (480, 1110, 100, 320).

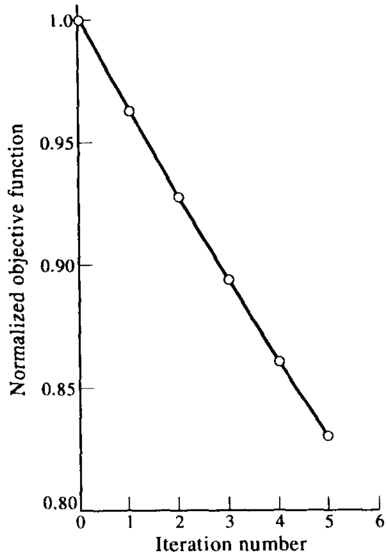

Fig. 3. Progress of optimization using SLP for trough type folded plate,

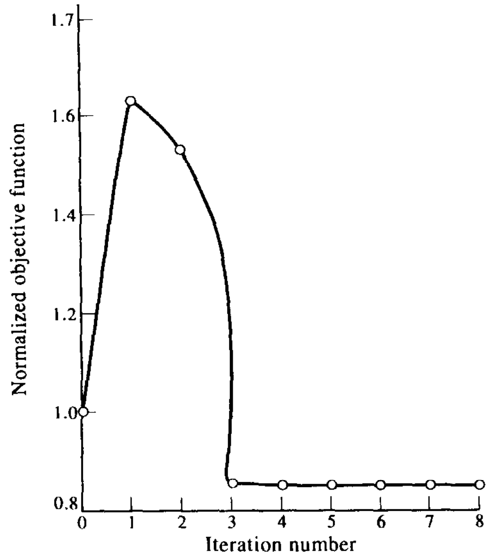

Fig. 4. Progress of optimization using SUMT for trough type folded plate.

PROGRESS OF OPTIMIZATION WITH SLP

The progress of optimization is shown in Fig. 3. The initial move-limits for the design vector have been taken as (20 mm, 30 mm, 10 mm and 2°). The optimization progressed smoothly without entering infeasible region and without any increase in objective function for the first five iterations. In the sixth iteration the linear programming solution at point number five with a normalized objective function value of 0.8316, resulted in a slightly higher value of the objective function (0.8332) which could not be reduced further. The optimum design vector is found to be (430.0, 1065.0, 75.0, 320.0). The percentage reduction in the objective function is found to be 17.6. The execution time is 10 s.

CONVERGENCE TO GLOBAL OPTIMUM USING SLP

Convergence to global optimum has been studied by using different starting points keeping all other parameters the same. The results are shown in Table 1. In the table the initial value of the design variable X_4 has been kept as 320 in almost all cases. This is because a number of preliminary runs have given the optimum value of X_4 as 320°. It can be seen that though the starting points are varied considerably, the optimum objective function values differ only slightly, indicating that a satisfactory optimum is attained. However, there is considerable difference in the values of design variables except for the thickness and inclination of the plate which are coming to their limiting values (75 mm and 320 degrees, respectively).

PROGRESS OF OPTIMIZATION WITH SUMT

The progress of optimization using SUMT is shown in Fig. 4. In the first unconstrained minimization of the P-function, there has been an increase in the value of the objective function and it has been reduced in the subsequent minimizations. It has also been observed that there is no significant improvement in the objective function after the fourth minimization. The optimum design vector is found to be

Table 1. Convergence to global optimum using SLP

Initial design point (X_1, X_2, X_3, X_4)	Optimum design point (X_1, X_2, X_3, X_4)	Optimum objective function $\times 10^{11}$	Optimum span–height ratio
480, 1110, 100, 320	430.00, 1065.00, 75.00, 320.00	0.2992	20.14
750, 1000, 120, 325	660.00, 985.00, 75.00, 320.00	0.2974	21.77
600, 900, 150, 320	450.00, 1095.00, 75.00, 320.00	0.3004	19.59
300, 1300, 200, 320	164.93, 1292.69, 75.02, 320.38	0.3119	16.82
600, 800, 250, 320	244.85, 1152.72, 75.00, 320.28	0.3039	18.79
528, 1072, 75, 320	422.64 965.42, 75.00, 320.00	0.2820	22.22

(535.58, 1159.6, 75.0, 320.67). Even for four minimizations it has taken nearly 3 h. The optimum objective function is found to be 0.3056×10^{11}. After several runs, since it has been found that SUMT is very slow in solving the optimum design problem of symmetrical trough type folded plates, it has not been used for further studies. All further studies have been carried out using SLP only.

Table 2. Effect of cost ratio—span = 18 m

Cost ratio	Optimum design variables X_1	X_2	X_3	X_4
80	430.0	1065.0	75.0	320.0
90	430.0	1065.0	75.0	320.0
100	430.0	1065.0	75.0	320.0
110	430.0	1065.0	75.0	320.0
120	430.0	1065.0	75.0	320.0

EFFECT OF COST RATIO

The effect of cost ratio on the optimum design of symmetrical trough type folded plate roofs is studied by varying the cost ratio from 80 to 120 in increments of 10. The results for a typical case has been shown in Table 2. It is observed that the optimum design variables are found to be independent of the cost ratio in most of the cases and the slight difference in some cases is not significant from practical point of view.

SELECTION OF INITIAL DESIGN VECTOR FOR FURTHER STUDIES

The study on convergence to global optimum stimulated the idea of deriving some guidelines for selecting initial design vector. For smooth progress and to get the best optimum objective function, it is better to start by keeping the width of top plate (X_1) and inner bottom plates the same. The initial thickness may be selected as 75 mm and γ_1 as 320°.

IS 2210 [1] has recommended to take the depth as span/15 for preliminary designs. The span–depth ratio is varied from 15 to 21 keeping all other parameters the same. It has been observed that keeping the span–depth ratio equal or close to 20 initially, gives a better optimum in many cases. Hence for further studies, while selecting initial design vector for a given span, a span–depth ratio of 20 has been taken.

EFFECT OF THE NUMBER OF TROUGH UNITS ON OPTIMUM DESIGN FOR A GIVEN SPAN

In practice, the overall dimension of a single trough unit will be the same for the entire width of the roof for a given span. The centre to centre distance between the inner bottom plates is considered as one trough unit. Studies were conducted for spans of 12, 15 and 18 m by keeping the overall dimension of a single trough unit as 3 m. In each case the numbers of trough units considered are 1, 2, 4 and 6. The optimum design variables along with the value of the objective function at optimum are given in Table 3.

It is observed that in certain cases, though there is slight variation in the values of X_1 and X_2, the optimum objective function is found to be proportional. This indicates that the entire number of trough units need not be considered for optimum design. Hence while preparing optimum designs, only two trough units have been considered for a given span.

OPTIMUM DESIGNS

In the design of trough type folded plate roofs, the overall dimension for a single trough unit is to be fixed based on the width of the roof to be covered. Generally, a width less than 3 m per single trough unit will be too low and a width more than 6 m per single trough unit will be too high. Optimum dimensions have been proposed for spans ranging from 12

Table 3. Effect of number of trough units for a given span on optimum design—overall dimension for single trough unit = 3 m

Span (m)	Number of trough units	Optimum design variables X_1	X_2	X_3	X_4	Optimum objective function for the width considered $\times 10^{-11}$
12	1	785.0	722.4	75.0	320.0	0.0721
	2	760.2	781.8	75.0	321.5	0.1452
	4	765.0	745.0	75.0	319.9	0.2904
	6	775.0	730.0	75.0	320.0	0.4344
15	1	596.0	879.0	75.0	320.0	0.1062
	2	596.0	879.0	75.0	320.0	0.2116
	4	596.0	879.0	75.0	320.0	0.4233
	6	596.0	879.0	75.0	320.0	0.6346
18	1	404.8	1041.1	75.0	320.0	0.1506
	2	484.7	1035.6	75.0	320.0	0.3011
	4	576.8	1033.7	75.0	320.0	0.6067
	6	592.1	1021.5	75.0	320.0	0.9085

Table 4. Optimum dimensions—span = 12 m

Overall dimension of single trough unit (m)	Optimum dimensions					Objective function per unit projected area in square square metre × 10^{-8}
	b_1 (mm)	b_2 (mm)	thickness (mm)	angle γ_1 (deg)	span-depth ratio	
3	760.2	676.3	75.0	321.5	19.29	2.017
3.5	1015.2	815.4	75.0	321.0	17.75	2.019
4	1135.0	985.2	75.0	320.0	15.21	2.071
4.5	1345.0	1155.0	75.0	320.0	14.30	2.122
5	1565.0	1345.6	75.0	320.0	13.68	2.124
5.5	1770.0	1505.1	75.0	320.0	12.85	2.104
6	1965.0	1645.7	75.0	320.0	11.97	2.068

to 24 m at intervals of 3 m. Only one table (Table 4) for a span of 12 m is given in this paper. For each span the width per trough unit ranges from 3 to 6 m in increments of 0.5 m. The load on the roof is as per IS 2210 [1]. The grade of concrete is M20 and that of steel is Fe 415.

The following are the observations made from the range of spans considered.

In all cases the optimum thickness is 75 mm and the optimum inclination of the plates with the horizontal is 40°. The ratio of span to width of a single trough unit is found to be between 4 and 5 for optimum design. If this ratio is high (i.e. for smaller unit width) the top plate width reduces, resulting in less concrete in compression zone. Hence the compressive stresses in concrete increase and the constraint on it becomes very active and results in higher thickness or no feasible solution.

CONCLUSIONS

Improved move-limit method of sequential linear programming is faster and efficient when compared with sequential unconstrained minimization technique for the optimum design of symmetrical trough type folded plate roofs. There is no considerable change in the design variables with change in cost ratio.

For a given span, it is enough to consider a minimum of two trough units for optimization. In general, from the above study, it is concluded that 75 mm trough type folded plate with 40° inclined plates (with horizontal) and with width per trough unit equal to 1/4 to 1/5th span is quite economical.

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