Minimization of maximum moment in offshore pipeline during installation

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During the laying of an offshore pipeline from a lay barge, the suspended length between the stinger end of the lay barge and sea bed is subjected to significant bending moments. The maximum moment in the pipeline depends on horizontal tension and the lift off angle at the stinger end. The problem of minimization of maximum moment in offshore pipeline is formulated by finding suitable values of horizontal tension and suspended length (and hence lift off angle) such that the maximum bending moment in the pipeline is as low as possible, without violating constraints on the lift off angle and geometric constraints. This is a nonlinear analysis and nonlinear optimization problem. For analysis Datta and Basu's formulation is used and the resulting nonlinear equations are solved by finite difference technique iteratively. The improved move limit method of sequential linear programming is used for optimization. In the present study only static loads are considered and optimum moments are found for one of the pipelines to be used in Bombay High. Progress of optimization, convergence to global optimum and suitability of the optimization technique used are discussed for a particular depth. Then optimum designs are obtained for various depths. It is found that by suitably adjusting the lift off angle the maximum moments can be considerably reduced. The optimum lift-off angle reduces with the depth.

Key Words: offshore pipeline optimization

INTRODUCTION

Since offshore oil production is on a large scale, the conventional method of transporting oil to land by tankers has become more expensive and laying pipeline is found to be economical. The offshore pipe is laid from a barge, which is a special ship provided with anchor winches, the pull winch and a crane. The lay barge is also provided with a stinger which helps to reduce the moment in the pipeline while laying. The pipe length AB (Fig. 1) suspended between the sea bed and the stinger end will be subjected to large moments due to static and dynamic loads. The static loads include the suspended weight of the pipeline, the horizontal tension etc. The dynamic loads include the drag force, the wave force etc. During laying of pipeline, the lift off angle β (Fig. 1) can be adjusted, but the length AB over which lift off takes place and the shape of deflected pipeline are unknowns. Since the shape of the pipeline depends on loading, it involves geometric non-linear analysis.

The analysis of suspended length shows that installation moment in the offshore pipeline depends on horizontal tension applied and the lift off angle. It is necessary to control these two parameters during laying of pipeline, otherwise unduly high moment is introduced which may cause breaking of the pipeline. If there is any breakage, the repairs involve a lot of expendture and delay in the progress of the work. Hence it is necessary to determine the optimum horizontal tension and lift-off angle for which the value of maximum moment in the pipeline is as low as possible. In the present work the authors have considered static loads only and have developed a computer program to get optimum design variables using finite difference technique for the analysis and sequential linear programming for the optimization. Minimizing the maximum moment in one of the offshore pipelines in Bombay High has been carried out.

MATHEMATICAL FORMULATION OF THE PROBLEM

A general constrained optimization problem may be stated as

$$\min Z = F(X) \tag{1}$$

subject to

$$g_j(X) \leqslant 0 \quad \text{where } j = 1, 2, \ldots, m$$

in which X = design variable of n dimensions, $F(X)$ = objective function, and $g_j(X)$ = constraints. Hence the mathematical formulation of optimization problem consists in identifying the objective function, design variables and the constraints.

The suspended length AB (Fig. 1) is subjected to heavy moments. This moment varies from point to point. The maximum moment governs the design. Minimizing this

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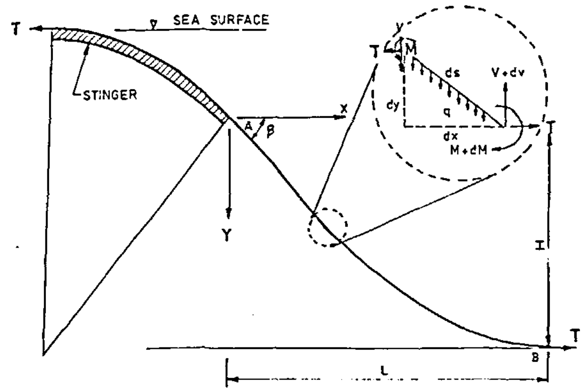

FIG. 1. SUSPENDED MARINE PIPELINE DURING LAYING.

maximum moment in the pipeline is taken as objective function.

The bending moment in the pipe depends on the horizontal tension T and lift off angle β which can be varied during installation. Hence these two can be taken as design variables. However in most of the methods of analysis available for the suspended pipeline, it is not possible to get the solution for known lift off angles directly. In these methods offshore pipeline of known suspended length is analysed and the lift off angle is calculated. A set of solutions give design charts from which the solution for the required lift off angles are obtained. Hence it is convenient to take horizontal tension T and suspended length L (Fig. 1) as design variables.

There are upper and lower limits for the lift off angle β within which the adjustments can be made. In the method of analysis, the value of β is obtained after carrying out the analysis for known suspended length of pipeline. Hence the limitations on lift off angle can be considered as behaviour constraints. Thus the following two constraints are to be imposed.

$$g_1(X) = \beta - \beta_u \leqslant 0 \tag{2}$$

$$g_2(X) = \beta_1 - \beta \leqslant 0 \tag{3}$$

where β_u = upper limit for the lift off angle and β_1 = lower limit for the lift off angle.

The following side constraints can be imposed to control the shape of suspended length:

$$g_3(X) = Y_{\max} - H \leqslant 0 \tag{4}$$

$$g_4(X) = Y_{\min} \geqslant 0 \tag{5}$$

where $Y_{\max}$ = maximum value of Y-coordinate, $Y_{\min}$ = minimum value of Y-coordinate, and H = depth of water from stinger end (Fig. 1). The upper limit on winch capacity is also to be considered as a design constraint. Thus

$$g_5(X) = T - T_{\max} \leqslant 0 \tag{6}$$

METHOD OF ANALYSIS

To get the objective function and behaviour constraint for a design point, it is necessary to analyse the suspended length of the pipeline. The following four types of analysis are available in the literature:

(i) Stiffened catenary method[1]
(ii) Finite difference method[2]
(iii) Finite beam segment method[3] and
(iv) Finite element method[4,5].

Except in the finite beam segment method, in all methods the analysis is carried out for known length AB and the lift off angle β is calculated. Then design charts are prepared to get the solution for known lift off angle β.

For analysis in the present investigation Datta and Basu's[2] formulation is used and the resulting equations are solved by finite difference technique. This formulation is an extension of the conventioal procedure for beam problem, except for the effects of geometrical nonlinearity included. In this method equations of static equilibrium are written down for an elemental length of the marine pipeline. Since it is a large bending equation the relation between the bending moment and the deflection to be used is

$$M = EI \frac{d^2y/dx^2}{(1 + dy/dx)^{3/2}} \tag{7}$$

instead of

$$M = EId^2y/dx^2 \tag{8}$$

where E is the Young's modulus, and I is moment of inertia of the section of pipe.

With this substitution the equilibrium equations can be reduced to a single fourth order differential equation in Y. The central finite difference technique can be used for solving the fourth order differential equation. This results in a set of nonlinear simultaneous equations in Y. Hence initially the shape of the pipeline is assumed as a second degree curve satisfying the boundary conditions. Linear simultaneous equations are formed and then solved by Gauss Jordon technique to get new values of vertical coordinate Y for all nodal points. If these values differ considerably from those in the previously assumed shape, the iterative procedure is continued. The details of this method may be found in reference 2.

METHOD OF OPTIMIZATION

For optimization any nonlinear constrained optimization technique can be used. For the present study the improved move limit method of sequential linear programming is used and is explained here briefly.

The Sequential Linear Programming consists in linearizing the objective function and the constraints in the vicinity of a design vector and solving the resulting linear programming problem to get a new design vector. The sequence of linearizing and solving the linear programming problem is continued from the new point till optimum is reached. The early suggestion of this method may be found in the work of Cheny and Goldstein.[6] When the optimum points do not lie at the corner of the constraint surfaces the oscillation of the design point is observed. One of the suggestions to overcome such situation is by Griffith and Stewart.[7] In the resulting linear programme, they imposed additional constraints to restrict the movement of the design variables. This move limit method has been used by Pope[8] for structural optimization. Some times, the linear programming solution gives a point where objective function is higher than at previous point. In such a case Pope[8] has suggested the interval halving technique to get a new design point. Whenever a linear programming solution gives an infeasible point, Pope has steered it is the feasible region by moving along the line joining the origin and the point.

Ramakrishnan and Bhavikatti[9, 10] suggested the following improvement in the move limit method to make it more powerful:

(i) When a linear programming solution gives a new design point, where objective function is higher, use quadratic interpolation instead of interval halving technique.
(ii) For steering an infeasible design point to the feasible region, move in the gradient direction of previous point instead of in the design vector direction.
(iii) After steering a design point to the feasible region, if it is found that the objective function is higher, there is need for the checking of the usability of the new direction instead of straight way going for quadratic interpolation. If the direction is not usable first carry out the quadratic interpolation between infeasible point and previous point and then steer the design point to feasible region, if necessary.

This improved method of sequential linear programming is used in the present investigation.

THE PROGRAM

A Fortran IV computer program is developed for minimizing the maximum moment in offshore pipeline during laying. The program may be subdivided into two parts, namely, optimizer and analyser.

Optimizer: It consists of a main program and three subroutines, namely, FUNCT, DERIV and SIMPLX. In the main program the following data is read:

(i) Number of variables
(ii) Number of constraints
(iii) Initial move limit
(iv) Lower and upper limits for the design variables
(v) Steplength for calculating derivatives by finite difference technique
(vi) Termination criteria.

The main program organizes the subroutines and linearizes the nonlinear objective function and constraints by using Taylor's series expansion. The resulting linear program is solved by using a simplex method in the subroutine SIMPLX. FUNCT is a small subroutine which calls the analyser directly or through DERIV, depending upon whether only functions are required or derivatives of the functions are required. DERIV is a subroutine for organizing the design derivative calculations by finite difference technique. After reaching optimum the details of the optimum design are printed out.

Analyser: The program developed for the analysis of stresses in offshore pipeline for the earlier work[11] has been modified to serve as analyser. When the analyser is entered first time, the following data is read:

(i) Inner and outer diameter of the pipe.
(ii) Submerged weight of the pipe.
(iii) Modulus of elasticity of the material of pipe.
(iv) Number of nodal points for finite difference technique.
(v) Depth of water, horizontal tension (initial) and horizontal length of suspended pipeline (initial).
(vi) Upper and lower limits for lift off angle.

The design derivative are defined. In the subsequent entries the reading of data and defining the design variables are skipped. Horizontal tension and suspended lengths are equated to the design variables 1 and 2 respectively. After the analysis the objective function and constraints are assembled.

RESULTS

The program developed is used to minimize the maximum moment in the pipeline which is going to be laid in the Bombay High region. The details of the pipe are as given below:

Outer diameter	0.914 m (36 in)
Inner diameter	0.857 m
Submerged weight	732.72 kg/m.

The maximum depth of sea in the region is 55 m. The modulus of elasticity of the material of the pipe is taken as 2.1×10^6 kg/cm^2. It is assumed that the lay barge will have a winch capacity of 200 tonnes and lift off angle can be varied from 5° to 50°.

For analysis, 21 nodal points are selected on pipe at equal horizontal distances. For 55 m depth of water, thorough investigation has been carried out to study the progress of optimization and convergence to global optimum. Then the program is used to determine the optimum design points for depth varying from 15 m to 55 m.

PROGRESS OF OPTIMIZATION

For a depth of 55 m, optimization is started with the point (110, 180 m). A move limit of ±2 t and ±5 m is imposed on the variables. The progress of optimization is shown in Fig. 2. The bending moment diagrams for initial and optimum design points are shown in Fig. 3. At optimum the values of maximum sagging and hogging moments are found to be almost same. At initial point, the hogging moment is maximum moment, its value being 1396.01 t-m. and occurs at first nodal point. After six iterations it came down to 739.23 t-m. This maximum moment occurred at 13th nodal point. After this iteration when the program solved the linear problem, the objective function is found to be higher and hence it does quadratic interpolation. It succeeded in reducing the moment of 13th nodal point and the move limit factor is also reduced proportionately. However, the bending moment at the neighbouring point is increased and that as an objective function the optimization continued. This slows down the progress of optimization. However, this situation arises when the design point is very close to optimum. A physical termination by activating maximum number of iterations is made use of.

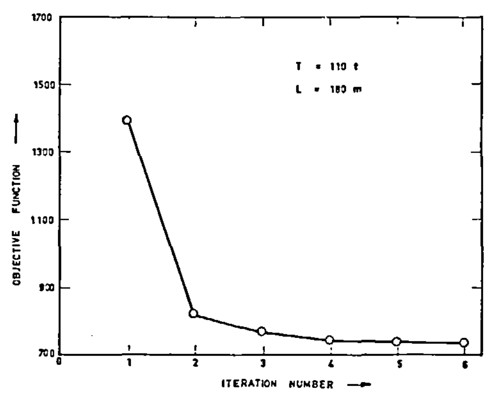

FIG. 2. PROGRESS OF OPTIMIZATION

STUDY OF CONVERGENCE TO GLOBAL OPTIMUM

To study whether optimization leads to global optimum, four different starting points are selected for 55 m depth. Table 1 shows the details of intial and corresponding optimum points. By observing the initial lift off angles obtained, it may be noted that there is considerable variation in the initial points selected for the study. In all cases, the optimum value of lift-off angle is nearly same, but the design variables, horizontal tension T and length L are very much different. This shows that the moment is very much sensitive to lift-off angle β and there are various combination of values of T and L, for which the same lift-off angle can be obtained. Most probably there is a single value of lift-off angle for which the optimum moment exists, but due to difficulties faced near the optimum, slightly different values are obtained.

OPTIMUM VALUES FOR VARIOUS DEPTHS

Since the lay barge method of laying pipeline cannot be used for a depth less than 15 m, studies are carried out for depths 15 m to 55 m at an interval of 10 m. The results obtained are shown in Table 2. The percentage reduction in maximum bending moment depends upon the initial point selected. It may be seen than in one of the cases, the reduction is as high as 74.85%. It shows the need for optimum studies in offshore pipe laying. The optimum lift-off angle reduces with the depth.

CONCLUSIONS

From the present study the following conclusions are drawn:

1. The improved move limit method of sequential linear programming leads to optimum point smoothly to a great extent. However, near the optimum if quadratic interpolation is required the progress of optimization is slowed down, since the point of maximum moment keeps on changing.
2. The moment in the pipeline is very much sensitive to lift-off angle. By adjusting the lift-off angle suitably the bending moment can be reduced considerably.
3. There are various combination of values of horizontal tension T and the span L for which the same lift-off angle can be obtained.
4. The optimum lift-off angle reduces with the depth.

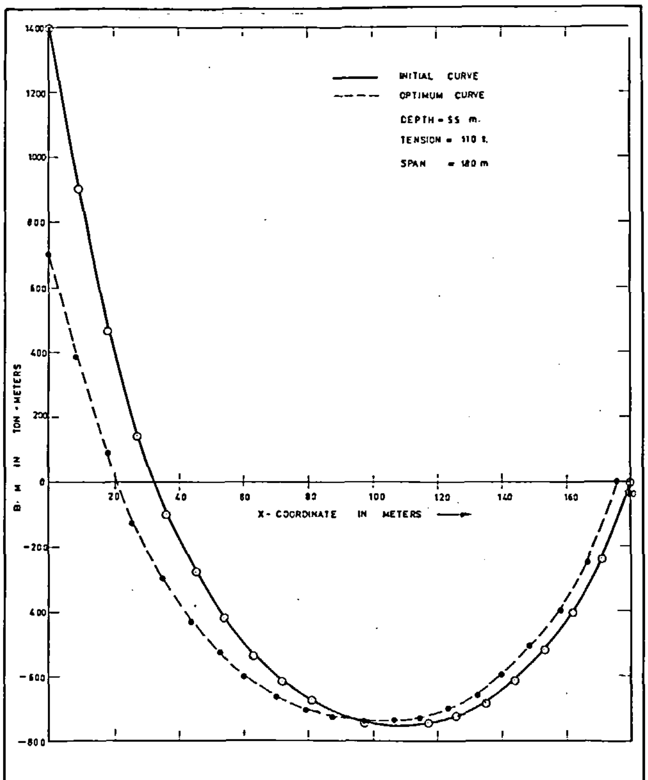

FIG. 3 BENDING MOMENT DIAGRAM

Table 1. Study of optimization from various starting points for the same depth of 55 m

Sl. No.	Tension in tonnes		Span in metres		Lift off angle in degrees		Max. B.M. in t-m		Percentage reduction in moment
	Initial	Optimum	Initial	Optimum	Initial	Optimum	Initial	Optimum	
1	110.00	116.98	180.00	175.82	23.02	30.30	1396.01	739.22	47.05
2	111.00	109.00	178.00	173.00	26.09	30.82	1111.57	767.52	30.95
3	120.00	124.41	165.00	176.02	49.47	32.94	791.63	720.10	9.04
4	75.00	85.39	225.00	164.17	5.39	32.73	3092.09	857.48	72.72

Table 2. Study of optimization for various depths

Sl. No.	Depth of water in metres	Tension in tonnes		Span in metres		Lift off angle		Max. B.M. in t-m.		Percentage reduction in B.M.
		Initial	Optimum	Initial	Optimum	Initial	Optimum	Initial	Optimum	
1	45	80.00	99.21	160.00	155.63	21.58	27.95	1301.18	808.08	37.90
2	35	55.00	52.53	135.00	137.49	26.21	23.07	874.80	858,60	1.85
3	25	40.00	46.00	140.00	125.00	5.23	17.84	1958.37	770.95	60.64
4	15	15.00	7.00	60.00	97.56	46.81	16.88	1999.59	754.44	74.85

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