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GRAPHICAL SYNTHESIS OF THE *RSSR* CRANK-ROCKER MECHANISM

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Abstract—Four different graphical constructions are put forward for the synthesis of the *RSSR* crank-rocker mechanism for prescribed oscillation angle and quick-return ratio. All these graphical constructions exhibit a series of designs, enabling some amount of direct choice. Checks for crankrocker action and for oscillation between the prescribed positions and in the prescribed direction are included along with a consideration of the transmission angle. These graphical procedures are believed to be useful when critical demands are not being made or while deviating somewhat from a limited set of catalogued designs.

1. INTRODUCTION

Graphical techniques of linkage synthesis, even when the constructions are not involved, have to contend with the tedium and time consumption of repeated construction necessary with variation in value of a free design parameter or two, in order to satisfy design constraints. Time and again, however, it is found convenient to follow a graphical approach when the demands are not critical or when slight modifications are desired from a limited set of catalogued designs.

Four different graphical constructions are put forward in the present work for the synthesis of the *RSSR* crank-rocker mechanism for prescribed oscillation angle and quick-return ratio (time ratio). The techniques described here are sufficiently simple and it is also possible to a considerable extent to exhibit a series of solutions by varying a parameter or two.

The associated problems of ensuring crank-rocker action, oscillation between prescribed positions and in the prescribed direction, and a minimum transmission angle are considered. A simple check for oscillation in the prescribed direction is put forward, not involving any additional construction.

A previous approach to graphical synthesis[1] is confined to the particular case of 90° shaft angle and deals with the three cases of prescribed axis distance, coupler length or crank length. Another work[2] utilizes trial and error procedures.

2. SOLUTION 1

A pictorial view of the *RSSR* crank-rocker mechanism in its limit positions 1 and 2 is shown in Fig. 1. A^*A_0 and B^*B_0 are the input and output shaft axes respectively and A^*B^* is the common perpendicular between them. A, B are the input side and output side spheric pair centres (A_1, B_1 in position-1, e.g.) and A_0, B_0 are the feet of the perpendiculars from A, B on the input and output axes. The dimensions of the mechanism are denoted as follows: input crank length, $A_0A = a$, coupler length $AB = c$, output rocker length $B_0B = b$, axis distance $A^*B^* = d$, axis angle $= \delta$, input side off-set $A^*A_0 = a_0$ and output side off-set $B^*B_0 = b_0$. A convenient fixed coordinate system $Oxyz$ is also included in Fig. 1, with O coinciding with A^*.

Figure 2 is common for all the four solutions described. There are thus a few extra lines while any one solution is being considered. It is believed that clarity is still maintained.

View (a) of Fig. 2 shows the two limit positions as viewed along xO (i.e. xA^*). The condition for a limit position (for a stationary position, more precisely) of the rocker B_0B is that B should lie in the plane formed by the point A and the axis A^*A_0.

It is convenient to introduce at this stage a condensed terminology regarding such planes and axes (Fig. 1):

(i) The input axis A^*A_0 is referred to as the A_0-axis and B^*B_0 as the B_0-axis.

(ii) The plane containing point A and the A_0-axis is called the A-plane. Thus we have the A_1 and A_2 planes for the limit positions 1 and 2. B_1 and B_2 planes can be similarly understood.

(iii) The plane of movement of A is normal to the A_0-axis and passes through A_0. It is referred to as the A_0-plane. The plane of movement of B is the B_0-plane.

Reverting to Fig. 2, B_1 and B_2 in view (a) lie on the A_1 and A_2 planes respectively. View (b) is the plan

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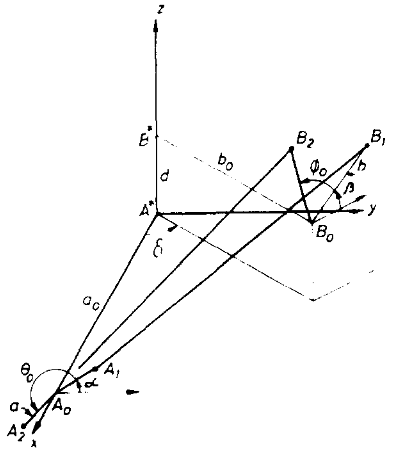

Fig. 1.

view of the mechanism in the two positions. It is obtained by looking along zO. View (c) is an auxiliary view obtained by looking along B_0B^*. View (d) is an auxiliary view obtained by looking normal to the A_1-plane from "above".

The following data are assumed (Fig. 1):

(i) The quick-return angle θ_0 and the oscillation angle φ_0. These are the specifications.

(ii) Location of the input and output axes. Hence the points A^*, B^* and the axis angle δ. The axis angle is usually specified.

(iii) Angles α, β, i.e. the crank and rocker position angles in position 1 of the mechanism.

The construction is carried out in the following steps.

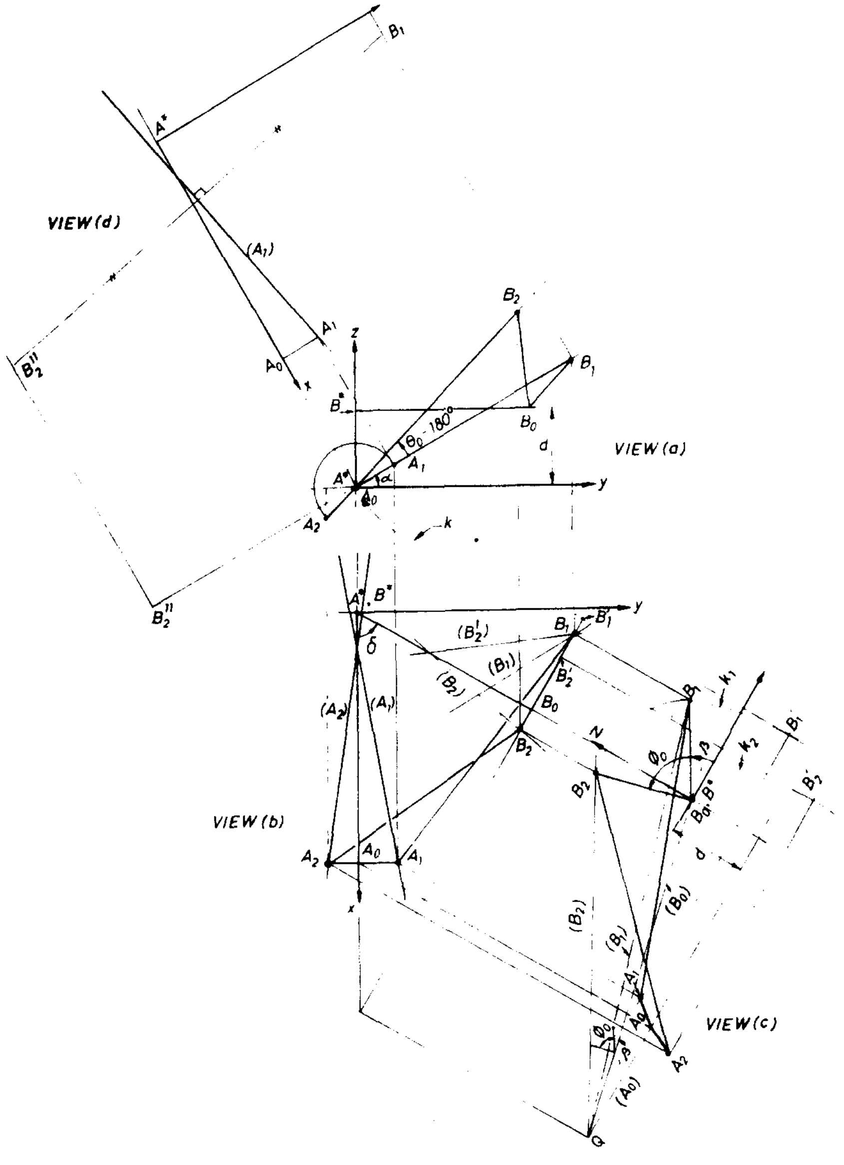

Fig. 2.

Part A

(1) Using α and θ_0, lines $A_0A_1B_1$ and $A_0A_2B_2$ in view (a) are drawn.

(2) Using δ and d, point B_0 in view (c) is located.

(3) Using β and φ_0, lines B_0B_1 and B_0B_2 in view (c) are drawn.

(4) Locus of B_1 is the line of intersection of plane $A_0A_1B_1$ (view a) and plane B_0B_1 (view c): it is thus the line $A_0A_1B_1$ in view (a) and line B_0B_1 in view (c). This line is now projected on the plan view (b) (marked as (B_1), i.e. locus of B_1).

(5) Locus of B_2 in the plan view (b) is similarly obtained. This is shown as (B_2), just like (B_1).

(6) Point B_2^1 is defined as the point obtained by rotating B_2 about B_0-axis through $-\varphi_0$ so that it falls in the B_1-plane. We obtain the locus of B_2^1 in the plan view (b) from that of B_2 by carring out the rotation in the auxiliary view (c). The construction lines are indicated in the figure.

(7) B_1 is obtained in the plan view (b) as the intersection of the loci of B_1 and B_2^1. B_2 can also be located now, using the auxiliary view (c).

(8) B_1 and B_2 are marked in view (a).

Part B

(1) Locate B_2^{11} in view (a) by rotating the line $A_0A_2B_2$ in that view through the angle $-\theta_0$ about the A_0-axis. (A_2 would thus have come and coincided with A_1).

(2) B_1 and B_2^{11} are transferred to the auxiliary view (d).

(3) Perpendicular bisector to $B_1B_2^{11}$ in view (d) is the locus of A_1 in this view, i.e. all the points on this line are solutions for A_1. This locus is transferred to view (b).

We have now at our disposal a series of designs with different values of crank length a and corresponding values of the off-set a_0. This set of designs has the values of α and β constant.†

It may be noted that the solution given here is based on a given location of the input and output axes. This may be sometimes advantageous in directly constructing the design on the assembly or lay-out drawing of a machine. The following three solutions are not directly for given locations of the axes but can be resized and shifted to the prescribed location of shafts.

3. SOLUTION 2

This solution differs from the previous one in assuming the location of the B_0-plane, instead of the location of the common perpendicular A^*B^*. Accordingly it differs in part A of the construction. Part B remains the same.

Procedure for part A:

(1) Using α and θ_0, lines $A_0A_1B_1$ and $A_0A_2B_2$ in view (a) are drawn.

(2) Location of the B_0-plane in the plan view (b) is chosen.

(3) Lines of intersection of the B_0-plane with the A_1 and A_2 planes are obtained as (B_1) and (B_2) in the auxiliary view (c). The A_0-axis is indicated as (A_0) in this view and the above lines (B_1) and (B_2) meet on this line at the point Q.

(4) For given φ_0 and β, the proportions and orientation of the triangle $B_1B_0B_2$ are available. Without choosing the size of this triangle (i.e. without choosing the rocker length b), we can thus draw one such triangle with corresponding vertices on the lines (B_1), (B_2) in the auxiliary view (c). The locus of B_0 is then the straight line joining the third vertex to the point Q. This locus is marked as (B_0).

(5) If the axis distance d is prescribed, the corresponding level line can be marked in the auxiliary view (c). Its intersection with (B_0) will give the point B_0. If the rocker length b is prescribed, the corresponding B_0 can be found by constructing the triangle QRB_0 with $QR = B_2B_0$ and RB_0 parallel to (B_2). This triangle is not shown in the figure.

(6) B_0 is marked on the B_0-plane in the plan view (b) and the common perpendicular A^*B^* is located.

4. SOLUTION 3

This is perhaps the best of the four solutions given here, combining simplicity with exhibition of solution series at two stages.

The orientations of the A_0 and B_0-axes are chosen, along with b and β.

The plan view (b) of the B_0-plane and the points B_0, B_1 and B_2 in the auxiliary view (c) are known. These points are transferred to the elevation (a) with arbitrary level of B_0. The locus of A^* in this view is a circle k through B_1 and B_2, on which the chord B_1B_2 subtends the angle $\theta_0 - 180°$. When the location of A^* has been chosen in this view, we have the positions of the A_0 and B_0-axes in the plan view (b), so that the common perpendicular is located.

The locus of A_1 can now be found as in part-B of solution 1.

It is also possible to obtain a prescribed value of axis distance d by marking the A^*y line accordingly in elevation (a).‡

This solution may also permit a rough judgement of design choice with respect to the transmission angle.

The three solutions considered till now allow choice of α and β. This is an advantage since the range of variation of α and β can be considerably reduced as shown in a companion work published in the same journal[3].

5. SOLUTION 4

The following data are assumed to start with: (i) crank position α, (ii) crank length a, and (iii) location of point A_0 and the B_0-plane.

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†The problem imposes 3 equality conditions and the mechanism has 7 non-dimensional defining parameters. Four parameters can thus be specified (δ, α, β, a/d in solution-1) and the others determined.

‡The four non-dimensional parameters chosen can thus be δ, β, d/b and a/b in this case.

Following are the steps of construction.

(1) Mark the lines $A_0A_1B_1$ and $A_0A_2B_2$ in view (a).

(2) In the auxiliary view (c), determine the line of intersection (B_1) of the B_0-plane with the A_1-plane. Determine (B_2) similarly.

(3) Locate A_0, A_1 and A_2 in the three views (a), (b) and (c).

(4) Choose the coupler length $AB = c$ and determine the points B_1' and B_2' on the B_0-plane in the plan view such that $A_1B_1' = A_2B_2' = c$, in this view.

(5) Using the projected length of A_1B_1' on the B_0-plane as radius and A_1 as centre, draw a circle in the auxiliary view (c) to obtain a locus of B_1.

(6) This locus k_1 of B_1 intersects the line (B_1) in view (c) in B_1. B_2 is similarly found as intersection of k_2 and (B_2).

(7) Knowing B_1 and B_2, B_0 is located in view (c) from the value of φ_0, the angle of oscillation.

(8) The common perpendicular A^*B^* can now be located.

It may be noted that the lines (B_1) and (B_2) remain the same for different crank lengths a and different coupler lengths c. It is thus possible to exhibit a number of designs for comparison and selection.

There are two solutions of B_1 and two of B_2 under item (6) above, making up four combinations. The choice between the combinations can be based on the checks (b) and (c) discussed in Section 7 later.

6. SPECIAL CASE OF $\theta_0 = 180°$

This is the case of unit time ratio. It is not possible in this case to choose both α and β independently. Accordingly only solutions 3 and 4 can be used. In using solution 3, the following simplification occurs (Fig. 3). Join B_1B_2 to intersect the (A_0)-line in the point X, in the auxiliary view (c). The parallel to the B_0-axis in the plan view (b) through X meets the A_0-axis in the plan view in Y. The B_0-plane can now be drawn in the plan view through Y. If A_0 is chosen at Y, A_1 is particularly easy to find.†

7. ANALYSIS

The analysis should take care of the following.

(a) *The mechanism should be a crank-rocker*

The synthesis procedure already ensures that the output link has limit positions. To ensure that the input link does not have limit positions, the elliptic locus of the projection of A on the B-plane (Fig. 4) may be used[4, 5]. The A-ellipse should fall entirely within the coupler circle. Moreover the farthest distance r to the points on the ellipse from the centre B of the coupler circle gives the minimum transmission angle: $\cos\mu = r/c$. Alternatively a graphical position analysis may be carried out for several positions and the transmission angle found in view (c) (see item d below). A direct analytical check is available in[5].

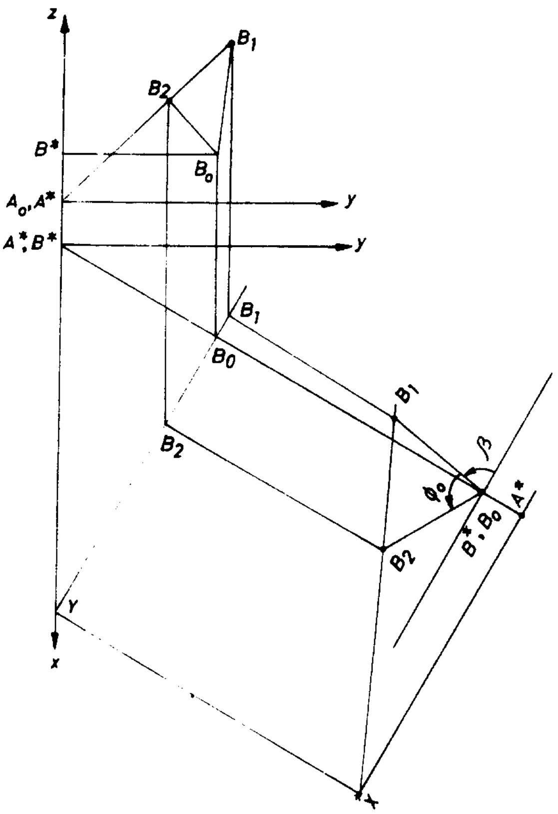

Fig. 3.

(b) *The oscillation should take place between the prescribed positions*

The prescribed limit positions B_1 and B_2 should fall on the same output branch. Otherwise the oscillation angle actually executed will be different from the desired one.

The branch can be checked by determining the transmission angles μ_1 and μ_2 along with their signs (μ-determination considered later).

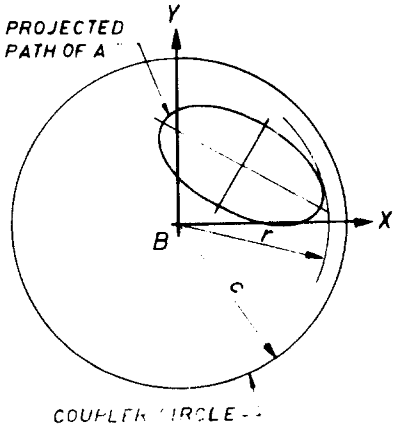

Fig. 4.

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†It is the intersection of two lines in view (c): (i) projection of line A_0A_1, and (ii) line perpendicular to XB_1 and distant $(\frac{1}{2})\mathbf{B_2B_1}$ from X (not shown in figure).

When once (and only when) it is established in (a) above that there are no limit positions for the input link, then the present check of demanding the same sign for μ_1 and μ_2 ensures that the oscillation takes place between these two positions.

The present check can however be advantageously applied even before crank-rocker action is established. Some designs are thus more easily eliminated.

(c) *The oscillation should take place in the prescribed direction*

There is nothing in the synthesis procedure or in the checks (a) and (b) above that can distinguish between the two alternative possibilites of oscillation, φ_0 and $\varphi_0 - 360°$ ($+150°$ and $-210°$, for example).

While it is possible to check for the direction of oscillation by making a position analysis for an intermediate position or by determining the direction of acceleration of B_1, a rather simple check is put forward below, that can be applied on the linkage construction already made, in one of the limit positions.

Before the final rules are given, some amount of explanation is necessary. Imagine B_1 fixed in its present position. The point A_1 is given a virtual displacement consistent with the constraints, except that A_0A or AB is allowed to change in length. Referring to the auxiliary view (d) of Fig. 2, it will be seen that the component of AB parallel to the A_0-axis does not change since A is still constrained to move on the A_0-plane. It is thus sufficient to consider the changes in length observed in view (a), parallel to the A_0-axis.

The following rules can be stated:

(i) If the point A_1 lies between A_0 and B_1 in elevation (a), the point B_1 is subjected to a pull along $\mathbf{B}_1\mathbf{A}_1$ (i.e. along the vector $\mathbf{B}_1\mathbf{A}_1$ in space and not just in view (a)).

(ii) If the point A_1 lies outside the stretch A_0B_1 in elevation (a), the point B_1 is subjected to a pull along $\mathbf{B}_1\mathbf{A}_1$ when A_1B_1, *in view* (a), is shorter than A_0A_1 and to a push along $\mathbf{A}_1\mathbf{B}_1$ when A_1B_1 is longer than A_0A_1 in view (a).

The language of these rules is based on the conception that B_0B is the driven link. The rules are analogous for position 2. They are applicable only to the two limit positions.

(iii) We now proceed to the auxiliary view (c) and utilize the knowledge of the direction of the coupler force to determine the direction of movement from B_1 (or B_2).

An illustrative example is provided by Fig. 2: A_1 lies between A_0 and B_1 in view (a). B_1 is thus pulled in view (c). The oscillation angle is thus the prescribed φ_0 and not $\varphi_0 - 360°$. It may be noted at this juncture that the $RSSR$ mechanism is capable of providing oscillation angles very much greater than 180° with reasonable transmission angles.

The series of solutions represented by the locus of A_1 in solutions 1, 2, 3 offers the possibility of integrating the checks (b) and (c) with the synthesis. This possibility has not been worked out.

(d) *Determination of the sign of the transmission angle and the minimum transmission angle*

The analysis is advantageously carried out with the position of B assumed: both the positions of A thus obtained will be valid. The distance of A from BB_0 in view (c) is $c \sin \mu$. To determine the sign of $\sin \mu$, look in every position from B_0 towards B and determine whether A is to your left or right.

In conclusion, it may be mentioned that checks (b) and (c), being simple and not needing additional construction, should be carried out first and only when they are satisfied it is necessary to check for the absence of limit positions of the input link and find the minimum transmission angle.

It may moreover be noted in passing that there is no need to determine the direction of movement from B_2 also (in addition to B_1) in check (c): when check (b) has already been carried out and found satisfied, the movements from B_2 and B_1 are necessarily opposed. This can be seen by considering the elliptic locus of projection of B on the input crank plane A^*A_0A.

8. CONCLUSION

Graphical solutions have been put forward for the synthesis of the $RSSR$ crank-rocker mechanism for prescribed rocker oscillation angle φ_0 and prescribed time ratio $\theta_0/(360 - \theta_0)$ where θ_0 is the crank rotation during one of the half-oscillations of the rocker. All these constructions assume at the outset the shaft angle δ. Since in co-ordinating angular displacements, only the ratios of linear dimensions are significant and not the absolute values, the various solutions can be viewed as follows. Solution 1 has a/d chosen at the end to exhibit a series of designs, solution 2 has a/d *or* a/b for the same purpose and solution 3 has a/b *and* possibly d/b. For solutions 1 and 2 (and for solution 3, if d/b is not chosen), the two remaining choices are the limit-position orientations of the crank and rocker. Solution 3 is perhaps the best. Apart from the axis angle δ, the limit-position orientation of the rocker is chosen at the outset. Choosing a value of d/b at the *last* stage of construction, a series of solutions is exhibited, described by a variation of a/b. Solution 4 chooses δ, α, $b_0 - a_0 \cos \delta = c_0$ (say), a and c (i.e. δ, α, a/c_0, c/c_0).

The $RSSR$ crank-rocker provides oscillation angles greater than 180°, bordering on 360° (as shown in a companion work communicated to the same journal). It is thus necessary to check that the oscillation between the limit positions (prescribed relative to each other) occurs in the prescribed direction. A simple check by inspection without additional construction is put forward in the present work for the purpose.

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GRAPHISCHE SYNTHESE DER RSSR-KURBELSCHWINGE

K. Lakshminarayana und L. V. Balaji Rao

Kurzfassung - Vier verschiedene graphische Verfahren werden für die Synthese der RSSR-Kurbelschwinge mit vorgeschriebenen Totlagenwinkeln, d. h. Schwingenwinkel und Zeitverhältnis, vorgestellt. Alle diese graphischen Verfahren liefern Lösungsserien, aus denen die direkte Auswahl einer für die Konstruktion geeignete Lösung möglich ist. Die Nachprüfung, ob tatsächlich eine Kurbelschwinge vorliegt und die Bewegungen zwischen den vorgeschriebenen Lagen und in der vorgeschriebenen Richtung erfolgen, wird eingeschlossen. Der Übertragungswinkel wird ebenfalls beachtet.