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Flexural Behaviour of Pretensioned Concrete Beams with Limited Prestress

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The behaviour of class-3 type prestressed concrete beams, at the limit states of cracking, deflexion and collapse are investigated by experiments on pretensioned beams with mild steel as supplementary reinforcement. Several methods of calculating the width of cracks are examined in the light of experimental results and an empirical formula, which includes the effect of percentage of untensioned reinforcement is suggested.

The deflexions of beams, based on the method of Beeby and Taylor, were marginally conservative when compared with the experimental results. The ultimate moment capacity of concrete sections with tensioned and untensioned reinforcement, was underestimated by as much as 10, 15 and 25 per cent by the Indian, British and American Code recommendations.

INTRODUCTION

THE PHILOSOPHY of design termed "Limit state approach", adopted by the Russian Code in 1954 is being adopted slowly by other National Codes. The influence of this design approach is evident in the revised American Code[1] and the British draft unified code[2]. Limit state design of prestressed concrete structural elements requires a proper knowledge of the behaviour of the members at the multiple limit states of cracking, deflexion and collapse. The use of limited or partial prestressing of concrete structures in which cracks of limited width are acceptable under occasional over loads or even under working loads, is embodied in the recent CEB–FIP recommendations[3]. The purpose of this investigation is to study the load-deformation, cracking and strength characteristics of pretensioned beams with different degrees of prestress and with different percentages of supplementary reinforcement.

Test-specimens

Flexural tests were made on beams with a rectangular cross section, 100 mm wide by 200 mm deep, over an effective span of 2·76 m. The details of pretensioned wires and supplementary reinforcement used in beams are given in Table 1 and figure 1. Two beams were fully prestressed and for the remaining six beams the prestress was varied from 3·0 to 8·6 N/mm^2. Mild steel bars conforming to the Indian Standard IS:432[4] with a guaranteed yield stress of 260 N/mm^2 were used as supplementary reinforcement. The prestress in the concrete was obtained by using 5 mm dia high tensile wires having a guaranteed 0·2 per cent proof stress of 1500 N/mm^2 and an ultimate tensile strength of 1650 N/mm^2.

In the prestressed beams of each group the degree of prestress was varied by controlling the number of high tensile wires and the flexural strength of all the beams was maintained approximately constant by using the required amount of supplementary reinforcement in the form of mild steel.

The beams were cast in a column type pretensioned bed fabricated in the laboratory. The concrete for the beams was mixed in the proportions of 1:0·72:2·18 by weight with a water–cement ratio of 0·38 using ordinary portland cement, together with sand and crushed granite aggregate of 20 mm, maximum size. The beams were cured by covering them with wet burlaps for 28 days before the start of tests. The average compressive strength of concrete recorded by testing 10-cm cubes was 42 N/mm^2.

INSTRUMENTATION AND EXPERIMENTAL PROCEDURE

The prestressed beams were tested using a loading frame and a 10 tonne hydraulic jack in conjunction with a proving ring for loading in small increments. The beams supported on rocker and roller bearings were loaded at one third points of the effective span. Batty dial gauges were used for recording the deflexions at regular load intervals.

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Table 1. Details of test beams

Beam number	Nominal effective depth (mm) d_{ts}	Nominal effective depth (mm) d_{us}	Number of 5 mm dia. tensioned wires	Supplementary reinforcement mild steel rods	Percentage of untensioned reinforcement	Approximate prestress at soffit of beam (N/mm²)
FP-1A FP-1B	137	180	5	—	—	11·0
LP-1A LP-1B	139	180	4	6 mm dia. 2 Nos.	0·3	8·6
LP-2A LP-2B	143	180	2	10 mm dia. 3 Nos.	1·2	5·5
LP-3A LP-3B	161	180	1	12 mm dia. 2 Nos. and 8 mm dia. 1 No.	1·6	3·0

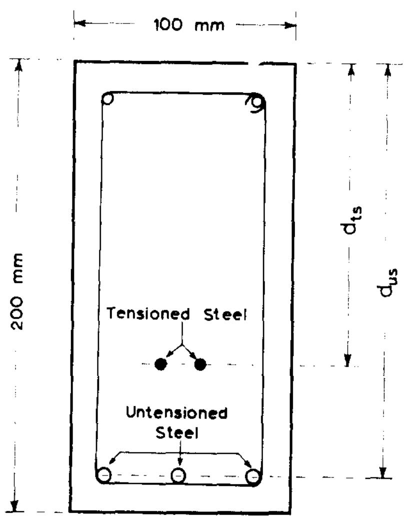

Fig. 1. Cross section of test beams.

Strain measurements were made for the central section of the beam by using a 20 cm demec gauge developed by Morice and Base[5]. Stainless steel demec targets were fixed on the surface at regular intervals to study the strain distribution across the central section of the beam. The cracks developed on the surface of the beams, and were observed through a Begg's microscope. The smallest width of cracks measurable with the instrument being 0·0015 mm. The maximum crack widths were measured at loading increments of 400 kg.

DISCUSSION OF TEST RESULTS

Cracking characteristics

The influence of supplementary reinforcement on the cracking characteristics of beams with varying degree of limited prestress is detailed in figure 2. The crack width behaviour is presented for increasing applied loads expressed as a function of the calculated ultimated load according to IS:1343[6]. The crack width at design load is considerably affected by the percentage of untensioned reinforcement used in the section. Increase of supplementary reinforcement significantly reduces the width of cracks at design loads. In the case of class-3 prestressed beams, Bennett and Chandrasekhar[7] have reported increased crack widths

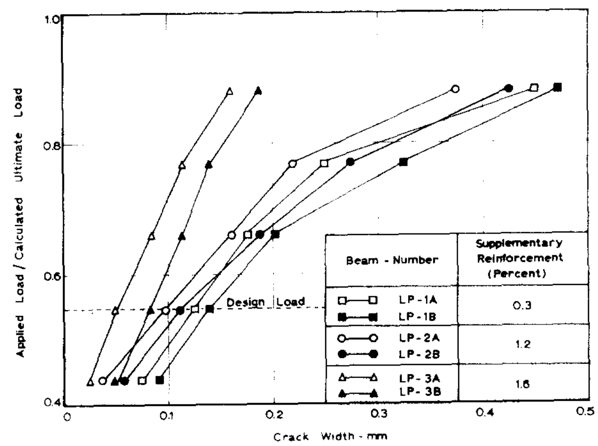

Fig. 2. Influence of supplementary reinforcement on load-crack width relationship.

when smaller area of steel is used as reinforcement. The draft unified code also recognises the importance of percentage of untensioned reinforcement in the cross section in limiting the allowable fictitious tensile stresses in the beam.

Prediction of crack width

In the limit state design, local damage is defined by a maximum allowable width of crack. Consequently the method of calculating crack width is of considerable importance. Methods suggested by Goschy[8], Borges[9], Birkenmeier[10], Baus[11] and the recent CEB–FIP report, relate crack width to the stress in the reinforcement. Abeles[12] has used the concept of fictitious tensile stress in concrete to predict the width of cracks, and the British draft unified code recommendations are based on this approach.

The reinforcement–stress method, although inherently more accurate for crack width predictions, has the disadvantage of lengthy calculations, whereas the fictitious tensile stress approach with its relative simplicity yields reasonably accurate values for practical purposes.

For the present series of tests on class-3 type pretensioned beams, the variation of crack width with fictitious tensile stress is presented in figure 3.

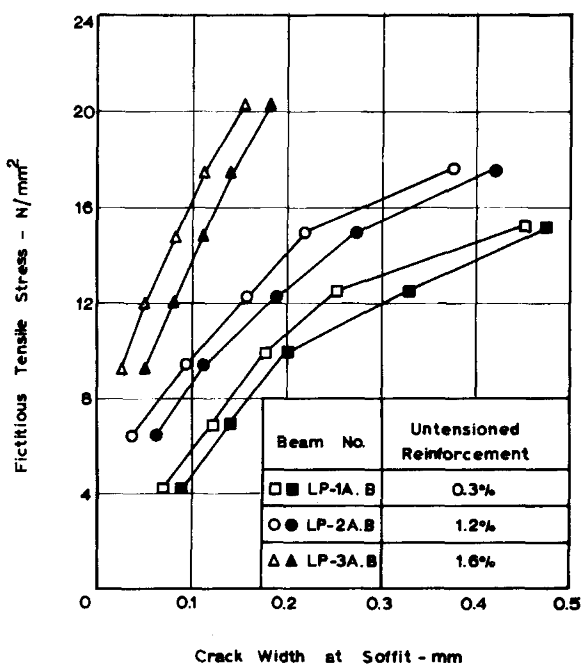

Fig. 3. Relation between crack width and fictitious tensile stress.

Based on the results of tests, it was found that the crack widths could be estimated by an expression of the type:

$$w = \frac{R}{P_e} \cdot C \cdot f_{ct}$$

where C is the cover over reinforcement, f_{ct} is the fictitious tensile stress in concrete, P_e is the percentage of untensioned reinforcement, and R is a factor depending upon the type of reinforcement. The experimental value of the constant R is 750×10^{-6} mm^2/N for mild steel bars, having a guaranteed yield stress of 260 N/mm^2.

In figures 4, 5 and 6, the experimental results are shown together with the recommended relations of Beeby and Taylor[13], Bennett and Chandra-

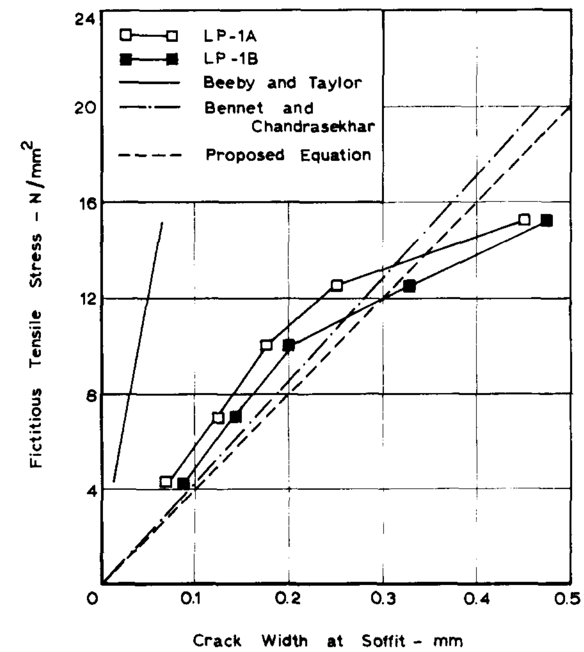

Fig. 4. Relation between crack width and fictitious tensile stress.

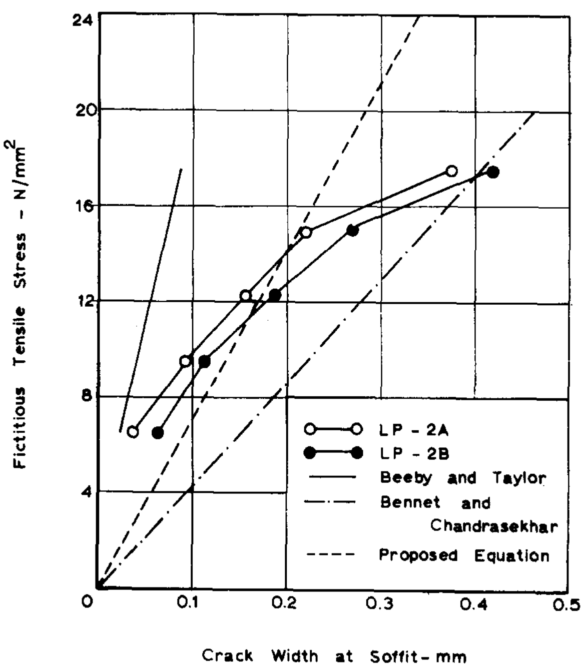

Fig. 5. Relation between crack width and fictitious tensile stress.

sekhar[7] and the proposed equation. The wide discrepancy between the different proposals is attributed to the different type of untensioned reinforcement used and the number of variables considered. The proposed equation incorporates the percentage of untensioned reinforcement as an additional variable, when compared with the other recom-

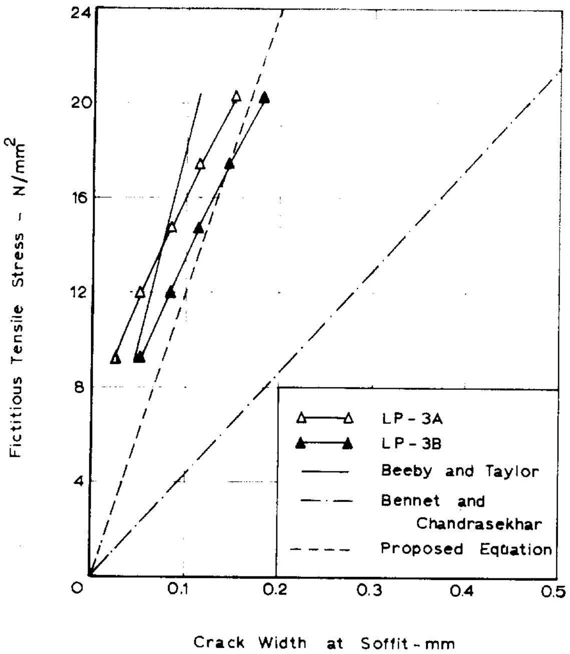

Fig. 6. *Relation between crack width and fictitious tensile stress.*

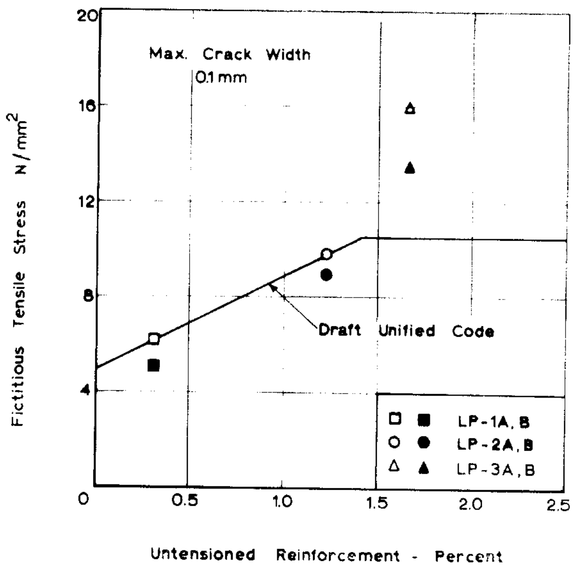

Fig. 7. *Relation between fictitious tensile stress and untensioned reinforcement.*

Table 2. *Results of maximum crack width at working loads*

Beam No.	Maximum crack width observed at 55 per cent of ultimate load (mm)	Calculated maximum crack width at 55 per cent of calculated ultimate load (according to IS:1343)		
		Beeby and Taylor	Bennett and Chandrasekhar	Proposed equation
LP-1A	0·12	0·02	0·16	0·17
LP-1B	0·14	0·02	0·16	0·17
LP-2A	0·09	0·04	0·22	0·14
LP-2B	0·11	0·04	0·22	0·14
LP-3A	0·05	0·06	0·28	0·10
LP-3B	0·08	0·06	0·28	0·10
Ratio of calculated to observed crack width	Mean	0·51	2·60	1·40
	Standard deviation	0·40	1·67	0·30
	Coefficient of variation (per cent)	78	64	21

mendations. The importance of the effective area of reinforcement on the prediction of crack widths has been reported by Kaar and Mattock[14].

Table 2 shows the results of measured crack widths at working loads corresponding to 55 per cent of the theoretical ultimate load, compared with the predictions of the various authors and the proposed equation.

The mechanism of crack formation is such that a wide scatter is inherent in the measured values of crack widths in tests and this is reflected in the statistics of the ratio of calculated to observed crack widths compiled in Table 2. In figures 7 and 8, the permissible fictitious tensile stresses for specified crack widths and percentage of untensioned reinforcement as recommended by the 1969 Draft British Code are compared with the results of present tests. From the plot, the code recommendations appear to be conservative for percentages of untensioned steel exceeding 1·5.

Deflexions of class-3 type prestressed beams

A knowledge of the moment–curvature relationship is essential to calculate the deflexions of partially prestressed members in which cracks are permitted under service loads. Investigations of moment–curvature relationships are reported by Beeby[15] in which a number of possible bilinear relationships are proposed and tested against experimental data. The moment–curvature relationship of each of the beams with limited prestress, was established by analyzing the cracked prestressed section on the lines suggested by Beeby and Taylor[13]. Theoretical load–deflexion curves shown in figures 9, 10 and 11 were derived by using an approximate method developed by Beeby and Miles[16], in which the maximum deflexion of the beam is expressed as a function of the maximum curvature, length and a constant which depends on the shape of the curvature diagram.

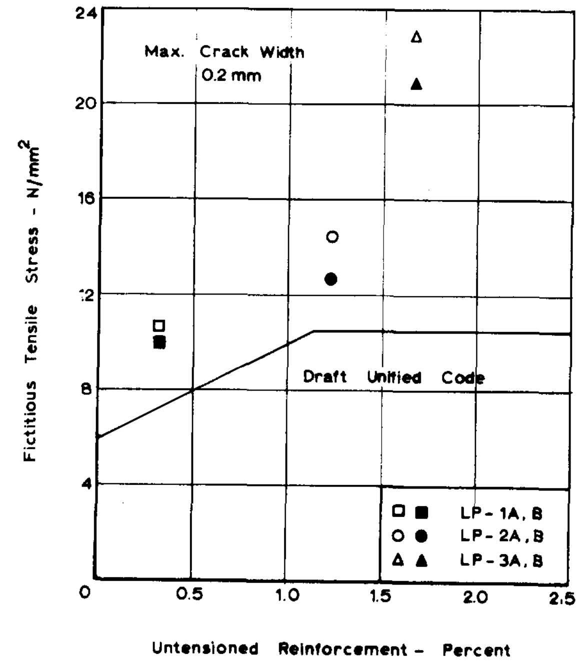

Fig. 8. Relation between fictitious tensile stress and untensioned reinforcement.

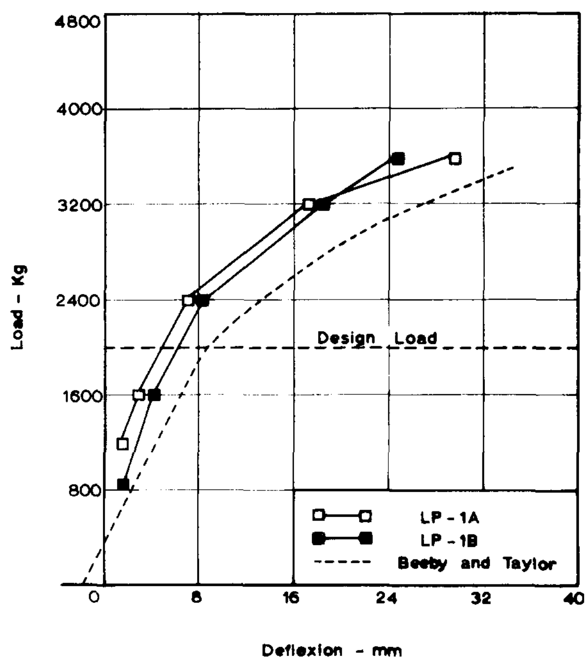

Fig. 9. Load–deflexion characteristics.

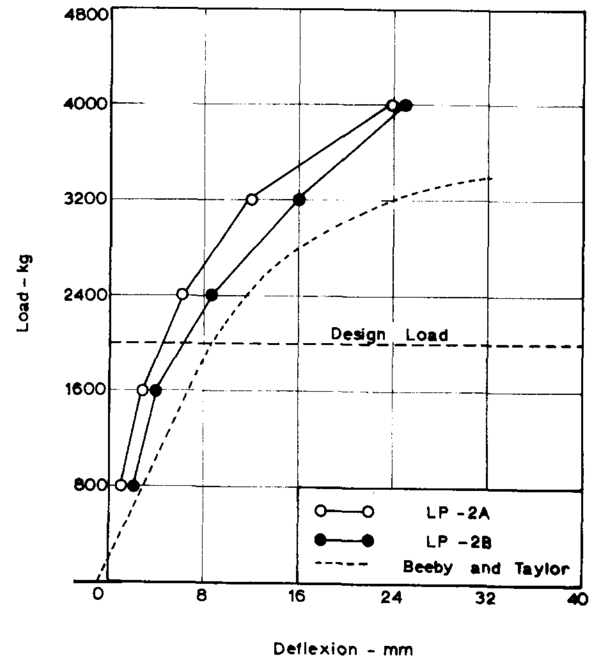

Fig. 10. Load–deflexion characteristics.

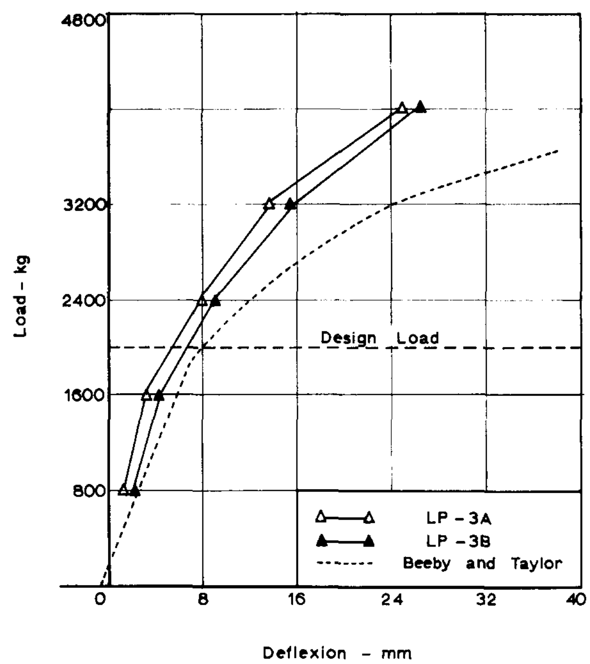

Fig. 11. Load–deflexion characteristics.

The theoretical predictions of deflexions of beams when tested against the experimental results in the figures has indicated that the theoretical estimates are consistently larger for all the three groups of beams and the difference being larger for loads over and above the design load. In Table 3, the theoretical and observed deflexions of beams at design load, corresponding to 55 per cent of the calculated ultimate load, are compiled. The ratio of the observed to the calculated deflexion varied from 0·56 to 0·82, indicating the conservative estimates of the theoretical procedure used in the computations.

Table 3. Deflexions of prestressed beams

Beam number	Deflexion at 55 per cent of ultimate load (mm)		Ratio of observed to calculated deflexion
	Observed	Calculated (Beeby and Taylor)	
FP–1A	5·0	—	—
FP–1B	4·0	—	—
LP–1A	5·0	9·0	0·56
LP–1B	6·0	9·0	0·67
LP–2A	5·0	8·5	0·59
LP–2B	6·5	8·5	0·77
LP–3A	5·5	8·0	0·69
LP–3B	6·5	8·0	0·82

The observed deflexions at design load are well within the limit of span/350 suggested by Bate[17] in the summary of basic requirements for limit state design and within the limit of span/360 proposed by the new ACI building code for floors not supporting or attached to non-structural elements likely to be damaged by large deflexions.

Ultimate moment capacity

The ultimate moment capacity of beams with full and limited prestress is compiled in Table 4, in which the observed moments are compared with the theoretical predictions based on the American, British[18] and the Indian Standard codes. The ratio of observed to calculated ultimate moment was never less than 1·0. The Indian, British and American code recommendations under-estimate the ultimate strength of prestressed beams used in this investigation by as much as 10, 15 and 25 per cent respectively. It is well established that the maximum steel stress at failure in prestressed beams is a function of the effective proportion of steel. A comparative study by Ramakrishnan[19] indicates that the ACI recommendations grossly underestimate the stress in steel at failure, compared to the British and Indian Codes which is reflected in the low values of the ultimate moment capacity of the cross section. However a more accurate method such as that presented by Warwaruk, Sozen and Siess[20] for calculation of stress in steel at failure would result in moments comparable to that of the observed values.

Table 4. Moment capacity of prestressed beams

Beam number	Observed ultimate moment (t.cm)	Ratio of observed to calculated ultimate moment using		
		IS:1343	CP:115	ACI–318
FP–1A	180	1·06	1·08	1·28
FP–1B	172	1·01	1·04	1·22
LP–1A	184	1·06	1·12	1·28
LP–1B	172	1·00	1·05	1·20
LP–2A	195	1·11	1·17	1·20
LP–2B	196	1·12	1·18	1·21
LP–3A	216	1·20	1·25	1·32
LP–3B	225	1·24	1·34	1·36

CONCLUSIONS

The following conclusions have been drawn by the study of the limit state behaviour of concrete beams with mild steel as supplementary reinforcement and limited prestress.

1. The percentage of untensioned reinforcement was found to have an important influence on the width of cracks.
2. The proposed expression for predicting the width of cracks accommodating the variables like the type and percentage of untensioned reinforcement, cover and fictitious tensile stress was found to give good correlation with the experimental results.
3. The formula proposed by Beeby and Taylor, underestimates the width of cracks in beams with mild steel as supplementary reinforcement.
4. The British draft code recommendations of permissible fictitious tensile stress for maximum crack width of 0·1 and 0·2 mm appear to be conservative for percentages of untensioned reinforcement exceeding 1·5.
5. The deflexions of beams, based on the method recommended by Beeby and Taylor were always on the safe side when compared with the experimental results.
6. The ultimate moment capacity of concrete sections with tensioned and untensioned reinforcement was underestimated by as much as 10, 15 and 25 per cent by the Indian, British and American Code recommendations respectively.

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Le comportement de poutres en béton pré-tensioné de la classe 3 aux états limites de cassure, de déflection et d'effondrement sont étudiés sur des poutres pré-tensionées avec de l'acier doux comme renforcement supplémentaire. Plusieurs méthodes de calcul suggérées pour la largeur des fentes sont examinées en vue des résultats expérimentaux et des formules empiriques, qui comprennent l'effet du pourcentage de renforcement non-tensioné.

Les fléchissements de poutres, fondées sur la méthode de Beeby at Taylor sont sensiblement conservateurs lorsqu'on les compare aux résultats expérimentaux. La capacité de moment ultime de sections en béton avec renforcement tensioné et non-tensioné, a été sous-estimée par 10, 15 et 25 pourcent même par les recommandations des codes Indien, Anglais et Américains.

Das Verhalten von Spannbetonträgern der Klasse 3 wird durch Versuche an Spannungsträgern mit Flusseisen als zusätzliche Bewehrung in den Grenzzuständen der Rissbildung, der Durchbiegung und des Zusammenbruchs untersucht. Mehrere Verfahren zur Berechnung der Rissbreite werden auf Grund experimenteller Ergebnisse untersucht, und es wird eine empirische Formel vorgeschlagen, welche die Wirkung prozentualer, nicht vorgespannter Bewehrung einschliesst.

Die Durchbiegungen von Trägern auf Grund des Verfahrens von Beeby and Taylor waren ziemlich mässig im Vergleich mit den experimentellen Ergebnissen. Die äusserste Momentenleistung von Betonabschnitten mit vorgespannter und nicht vorgespannter Bewehrung wurde bei den Indischen, Britischen und Amerikanischen empfohlenen Vorschriften bis auf 10, 15 und 25 Prozent unterschätzt.