Short Papers

Automated Multi-Agent Search Using Centroidal Voronoi Configuration

K. R. Guruprasad and Debasish Ghose

Abstract—**This paper addresses the problem of automated multiagent search in an unknown environment. Autonomous agents equipped with sensors carry out a search operation in a search space, where the uncertainty, or lack of information about the environment, is known *a priori* as an uncertainty density distribution function. The agents are deployed in the search space to maximize single step search effectiveness. The centroidal Voronoi configuration, which achieves a locally optimal deployment, forms the basis for the proposed *sequential deploy and search* strategy. It is shown that with the proposed control law the agent trajectories converge in a globally asymptotic manner to the centroidal Voronoi configuration. Simulation experiments are provided to validate the strategy.**

Note to Practitioners—**In this paper, searching an unknown region to gather information about it is modeled as a problem of using search as a means of reducing information uncertainty about the region. Moreover, multiple automated searchers or agents are used to carry out this operation optimally. This problem has many applications in search and surveillance operations using several autonomous UAVs or mobile robots. The concept of agents converging to the centroid of their Voronoi cells, weighted with the uncertainty density, is used to design a search strategy named as *sequential deploy and search*. Finally, the performance of the strategy is validated using simulations.**

Index Terms—**Autonomous agents, cooperative systems, distributed control, multiagent search, Voronoi partitions.**

I. Introduction

THE PROBLEM of searching for targets in unknown environments has been addressed in the literature in the past. Various search strategies available in the literature have been surveyed in [1]. These fundamental works were mostly theoretical in nature and were applicable to a single agent searching for single or multiple, static or moving, targets. Cooperative search by multiple agents has been studied by various researchers [2]–[10] using concepts such as predefined lanes or patterns, space filling curves, dynamic programming, distributed learning, game and team theory. Some researchers have associated the search space with an uncertainty density distribution which is reduced when an agent performs search (e.g., [11]). Another related concept is proposed by Cortes *et al.* [12], who use Voronoi partitions to solve a spatially distributed optimal deployment problem for multiagent systems. Centroidal Voronoi configuration is shown to be a locally optimal deployment of sensors.

In this paper, a Voronoi partition approach is used to design a multiagent search strategy. Each agent acts within its Voronoi cell where it is most effective. The agents are deployed so as to maximize their search effectiveness in each step when the search is performed. During search operation, the uncertainty density, which represents the lack of information, is reduced. Some preliminary results were earlier presented in [13].

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II. Sequential Deploy and Search (SDS) Strategy

There are N agents performing search operation in an unknown environment. The search space $Q \subset \mathbb{R}^d$ is a convex polytope in d-dimensional Euclidean space. The lack of information is modeled as an uncertainty density distribution $\phi : Q \mapsto [0,1]$. The configuration of agents at any given time t is $P(t) = (p_1(t), p_2(t), \ldots, p_N(t)) \in Q^N$, with $p_i \neq p_j$, whenever $i \neq j$, where $p_i(t)$ is the position of the ith agent at time t. Each sensor's effectiveness is assumed to be a strictly decreasing function of $\|p_i - q\|$, which is the Euclidean distance between the sensor location p_i and a point of interest $q \in Q$. The problem addressed in this paper is that of deploying N agents in Q to collect information, thereby reducing the uncertainty density distribution over Q.

After agents are deployed, they perform search to acquire information about the search space and update the uncertainty density distribution ϕ. The entire procedure of *deploy and search* continues until the density distribution at every point in the search space is below a threshold limit.

After deployment, the sensors gather information about Q, reducing the uncertainty density according to

$$\phi_{n+1}(q) = \phi_n(q) \min_i \{\beta(\|p_i - q\|)\} \tag{1}$$

where $\phi_n(q)$ is the uncertainty density at the nth *deploy and search* step; $\beta : \mathbb{R} \mapsto (0,1)$ is a strictly increasing function of the Euclidean distance from the agent, and acts as a factor of reduction in uncertainty by the sensors. At a given $q \in Q$, only the agent with the smallest $\beta(\|p_i - q\|)$, that is, the agent which can reduce the uncertainty by the largest amount performs search. If all the agents search within their Voronoi cells, then evaluating $\min_i\{\beta(\|p_i - q\|)\}$ in (1) is equivalent to evaluating $\beta(\|p_i - q\|)$, with $p_i \in V_i$, the Voronoi cell corresponding to the ith agent. Note that n represents the number of *deploy and search* instances, and not time. Here, $p_i = p_i(t)$ is the position of the ith agent at the time instance t at the end of the nth deployment step.

A. Objective Function

The deployment of the agents in Q should maximize the reduction in uncertainty ϕ in any given iteration. Thus, the following objective function is maximized:

$$\begin{aligned}\mathcal{H}_n &= \int_Q \Delta\phi_n(q) dQ = \int_Q (\phi_n(q) - \phi_{n+1}) dQ \\ &= \sum_i^N \int_{V_i} \phi_n(q)(1 - \beta(\|p_i - q\|)) dQ. \end{aligned} \tag{2}$$

The gradient of the objective function $\mathcal{H}_n$ with respect to p_i is given by (using generalized Leibniz Theorem [14])

$$\frac{\partial \mathcal{H}_n}{\partial p_i} = \int_{V_i} \phi_n(q) \frac{\partial}{\partial p_i}(1 - \beta(r_i)) dQ \tag{3}$$

where $r_i = \|p_i - q\|$. Note that the gradient given by (3) is spatially distributed over the *Delaunay graph* $\mathcal{G}_D$, where two agents i and j are

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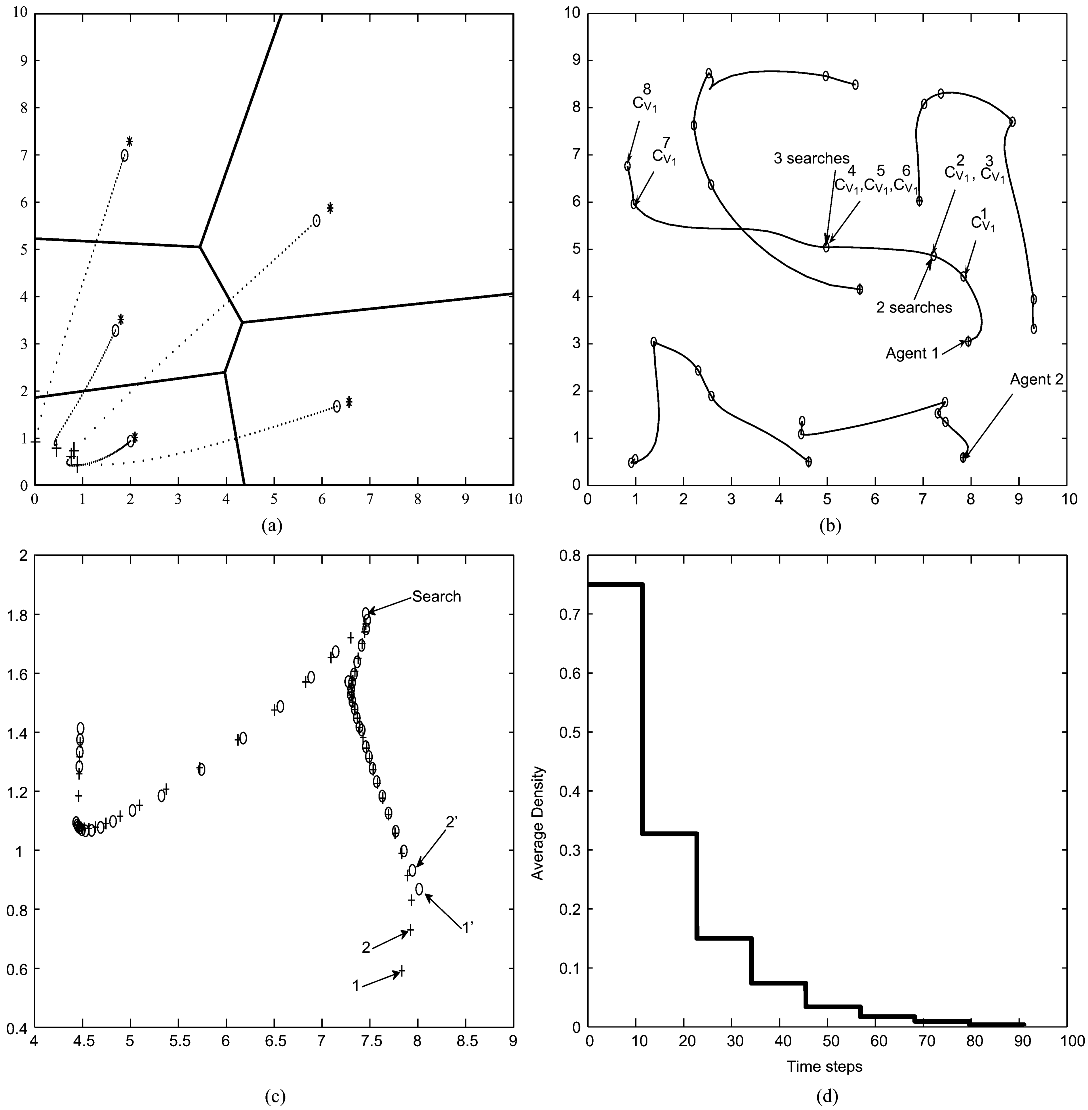

Fig. 1. (a) Agent trajectories during the first deployment step and the final Voronoi partition. (b) Agent trajectories for a complete search operation. (c) Trajectory of Agent 2 is shown to follow the centroid. (d) The reduction in average uncertainty density with time steps.

considered neighbors if and only if $V_i \cap V_j \neq \emptyset$. In the following, we use ϕ instead of ϕ_n for simplicity. The gradient (3) can be rewritten as

$$\begin{aligned}\frac{\partial \mathcal{H}_n}{\partial p_i} &= 2\int_{V_i} \phi(q)\frac{\partial f}{\partial ({r_i}^2)}(p_i - q)dQ \\ &= -\int_{V_i} \tilde{\phi}(q)(p_i - q)dQ = -\tilde{M}_{V_i}(p_i - \tilde{C}_{V_i}) \qquad (4)\end{aligned}$$

where $f(\cdot) = 1 - \beta(\cdot)$ and $\tilde{\phi}(q) = -2\phi(q)\partial f/\partial({r_i}^2)$, which can be interpreted as the density modified or perceived by the sensor. Here, the chain rule $\partial f/\partial p_i = \partial f/\partial(r_i^2) \cdot \partial(r_i^2)/\partial p_i$ has been used, and $\partial(r_i^2)/\partial p_i = 2(p_i - q)$. Now, f being a strictly decreasing function, $\tilde{\phi}(q)$ is always non-negative. Hence, $\tilde{M}_{V_i}$ and $\tilde{C}_{V_i}$ can be interpreted as the mass and centroid of the cell V_i with $\tilde{\phi}$ as density. Thus, the critical points are $p_i = \tilde{C}_{V_i}$, and such a configuration P of agents is called a *centroidal Voronoi configuration*. Note that $\tilde{C}_{V_i}$ depends on P, and so $p_i = \tilde{C}_{V_i}(P)$, $\forall i$, is a fixed point.

The objective function (2) is similar to that used in [12], which uses Voronoi partitions to formulate a multi-center objective function to maximize the coverage by multiple mobile sensors. A density function ϕ represents the probability of occurrence of an event of interest and a function f of the Euclidean distance provides a quantitative assessment of the sensing performance. Motivated by their approach, a multiagent search problem using Voronoi partitions is formulated in this paper where the objective is to maximize the search effectiveness. Though the form of the objective function remains the same, there are a few important differences in the formulation. The interpretation of ϕ and f are not the same as in [12]. The lack of information is modeled as an uncertainty density ϕ and f represents the sensor effectiveness. A centroidal Voronoi configuration is obtained as a local optimal configuration in each step of deploy and search operation, for any f which is strictly decreasing and continuously differentiable (unlike $f(\cdot) = -\|p_i - q\|^2$ in [12]), with a density modified/perceived by the sensor given by $\tilde{\phi}(q)$ (unlike with density ϕ in [12]). Finally, as mentioned earlier, this paper addresses a problem of multiagent deploy and search operation and subsequent information uncertainty reduction rather than a problem of optimal sensor coverage.

B. The Control Law

Consider the agent dynamics and control law as

$$\dot{p}_i(t) = u_i(t) \qquad (5)$$

$$u_i(t) = -k_{prop}(p_i(t) - \tilde{C}_{V_i}(t)). \qquad (6)$$

The control law (6) makes the agents move toward $\tilde{C}_{V_i}$ for positive control gain k_{prop}.

Theorem 1: The trajectories of the agents governed by the control law (6), starting from any initial condition $P(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}_n$.

Proof: LaSalle's invariance principle [15] is used here. Consider $V(P) = -\mathcal{H}_n$

$$\begin{aligned}\dot{V}(P) &= -\frac{d\mathcal{H}_n}{dt} = -\sum_i^N \frac{\partial \mathcal{H}_n}{\partial p_i}\dot{p}_i \\ &= -k_{prop}\sum_i^N \tilde{M}_{V_i}(\|p_i - \tilde{C}_{V_i}\|)^2. \qquad (7)\end{aligned}$$

Suppose the ith agent is located on ∂Q, the boundary of Q, at some time t. Let $L = \partial Q \cap V_i$ be a part of the bounding hyperplane common to ∂Q and V_i, containing p_i. Let $\hat{n}$ be unit outward normal to L. From (5) and (6), we have

$$\begin{aligned}\dot{p}_i &= -k_{prop}(p_i(t) - \tilde{C}_{V_i}(t)) \\ &= \frac{k_{prop}}{M_{V_i}}\int_{V_i}\tilde{\phi}(q)(q - p_i)dQ. \qquad (8)\end{aligned}$$

The inner product of $\dot{p}_i$ with $\hat{n}$ is given by

$$\hat{n}\cdot\dot{p}_i = k_{prop}\frac{\int_{V_i}\tilde{\phi}(q)\hat{n}\cdot(q-p_i)dQ}{\int_{V_i}\tilde{\phi}(q)dQ}. \qquad (9)$$

The quantity $\hat{n}\cdot(q - p_i) \leq 0,\ \forall q \in V_i$ (equality holds only for $q \in L$), and hence, $\hat{n}\cdot\dot{p}_i \leq 0$. That is, $\dot{p}_i$ points toward $\mathrm{int}(Q)$, the interior of Q, or is tangential to ∂Q. Further, suppose hyperplanes L_j, $j \in \{1,\ldots,M\}$ form an edge or a corner, and $p_i \in \cap_{j=1}^M L_j$. Then, $\dot{p}\cdot\hat{n}_j \leq 0,\ \forall j \in \{1,\ldots,M\}$, where $\hat{n}_j$ is the outward normal to L_j. Thus, Q is invariant under (8) (see [16, Th. 3.1]). The result also follows as $\tilde{C}_{V_i} \in V_i$. For another alternative proof, see [17, Lemma 2.8], which can be modified to suit the situation.

Observe that $V : Q \mapsto \mathbb{R}$ is continuously differentiable in Q as the Voronoi partition $\{V_i\}$ depends at least continuously on P (note that $p_i \neq p_j$, whenever $i \neq j$); Q is a compact invariant set; $\dot{V}$ is negative definite in Q; $E = \dot{V}^{-1}(0) = \{P|\tilde{C}_{V_i}(P) = p_i,\ \forall i\}$, which itself is the largest invariant subset by the control law (6). Thus, by LaSalle's invariance principle, the trajectories of the agents governed by control law (6), starting from any initial configuration $P(0) \in Q^N$, will asymptotically converge to the set E, the critical points of $\mathcal{H}_n$, that is, the centroidal Voronoi configuration with respect to the density as perceived by the sensors. □

Note that unlike Lyapunov stability principle, in LaSalle's invariance principle, the function V need not be positive definite (see the remark on Theorem 3.8 in [15, pp. 90–91]).

The control law (6) and the SDS strategy are spatially distributed over the Delaunay graph $\mathcal{G}_D$.

The centroid is computed based on the density information. The Voronoi partition is updated and the centroids are recomputed as the agents move. At the end of a deployment step, the control law (6) ensures that each agent is at (or sufficiently close to) the centroid of the corresponding Voronoi cell.

C. Some Practical Considerations

To implement the control law, the centroid of each Voronoi cell needs to be computed. The computational overhead of obtaining the centroid can be reduced at the cost of slower convergence using methods such as random sampling and stochastic approximation [18], [19]. In this work, the search space is discretized into grids while implementing the strategy. This simplifies the computation of centroids of the Voronoi cells. However, issues such as computational complexity are beyond the scope of this paper.

No constraints on the agent speed and limit on sensor range in the problem formulation have been considered. However, the control law can be modified to account for constant speed or maximum speed limit. In the case of limited sensor range, the entire space Q may not be accessible to the agents, and each agent can perform search within $V_i \cap \bar{B}(p_i, R)$, where $\bar{B}(p_i, R)$ is the closed ball of radius R, representing the sensor range, centered at p_i. The objective function can be modified to account for the limited sensor range. However, it would not be possible to ensure that SDS strategy successfully reduces the average uncertainty density to any arbitrary value in finite time. The details of the analysis under these constraints are not given here due to lack of space. However, in the simulation experiments carried out to validate the proposed search strategy, saturation on the agent speed has been considered.

Synchronization and spatial distribution are important issues in decentralized schemes such as the search strategy presented in this paper. It is assumed that initial uncertainty density is known *a priori* to all the agents. It can be seen that SDS strategy is spatially distributed over the Delaunay graph $\mathcal{G}_\mathcal{D}$. This implies that the agents do computations based only on information about positions of neighbors. Also, the agents should have access to the updated uncertainty map within their Voronoi cells. This can be achieved in several ways. One such way is that all the agents should communicate with a central information provider. However, it is not necessary to have this global information. Each agent i can communicate with its neighbors in the Delaunay graph $\mathcal{G}_\mathcal{D}$ ($\mathcal{N}_\mathcal{G}(i)$) and obtain the updated uncertainty information in a region $\cup_{\mathcal{N}_\mathcal{G}(i)} V_i$. As the Voronoi partition $\{V_i\}$ depends at least continuously on P in an evolving Delaunay graph, communication between neighbors is sufficient for each agent to obtain the uncertainty value within its Voronoi cell. Issues related to communication of uncertainty information are not addressed in the paper except to assume that uncertainty information is available to the agents.

Theoretically, all the agents reach the respective centroid at infinite time. However, in a practical implementation, the agents are required to be sufficiently close (where closeness is suitably defined) to the respective centroids before starting search operation. Agents need to come to a consensus as to when to end deployment and perform search. In a practical situation, synchronization can be attained by agents communicating a flag bit indicating if an agent has reached its centroid or not. When all the agents have reached their respective centroids within a tolerance distance, search is performed. It is also assumed that sensing and communication are instantaneous. In simulation tests, given in the next section, it is assumed that such communication links exist. Since the objective of this work is to evaluate the effectiveness of the search strategy, assumptions are made that simplify implementation without affecting search effectiveness. The strategy is implemented in a single centralized program using MATLAB. Further, in [12], the authors provide an asynchronous implementation for coverage control which can also be modified for SDS.

It can be shown that the SDS strategy can reduce the average uncertainty to any arbitrarily small value in finite time. This result does not depend on the control law, but depends only on the choice of the updating function (1), along with the fact that there is no sensor range limitation, and that the search space Q is bounded. In addition, it does not address the issue of optimality of agent trajectories which, in fact, depends on the control law which decides the motion of the agents. We do not provide bounds on the time required for reduction of average uncertainty density below a specified value. However, the reduction in uncertainty in each step in SDS is given by

$$\mathcal{H}_n^* = \sum_i \int_{V_i} \phi_n(q) f(\|\tilde{C}_{V_i} - q\|\|)dQ. \qquad (10)$$

III. Simulation Results and Discussions

The results of some of the simulation tests carried out to illustrate and validate the SDS strategy are presented here. A single step of deployment and search operation is referred to as one *step* or *iteration*. The parameters used are the following: Q is a 10×10 (distance units) square area in $\mathbb{R}^2$. Initial uncertainty density is a constant distribution of 0.75 over Q. A maximum speed for agents was fixed at 1 (speed unit). The controller gain $k_{prop} = 0.5$. An exponential function $\beta(r) = 1 - ke^{-\alpha r^2}$, with $k = 0.8$ and $\alpha = 0.1$ was used. The iterations were terminated when the maximum density over Q goes below 0.05. A discrete implementation of the control law (6) is used with time period of 1 (time unit).

Fig. 1(a) shows trajectories of 5 agents during a single deployment step of the proposed SDS strategy. The agents are deployed to the optimal configuration without executing search, and hence there is no change in the uncertainty density. The Voronoi partition changes as the agents move. The initial positions of the agents are shown by "+", the intermediate positions by dots, and the final positions by "o"s. The Voronoi partition for the agent configuration at the end of the deployment step is also shown, with "*" indicating the centroids. It can be observed that the final position of each agent is sufficiently close to the corresponding centroid.

In another example, five agents performing search using SDS strategy is considered. Fig. 1(b) shows the trajectories of agents until the maximum uncertainty density is reduced to a value below 0.05. The initial location of the agents are indicated by "+", and the points at which search is performed by "o". It can be observed that in this case eight deployment steps were required to reduce the uncertainty below the desired value. Points marked $C_{V_1}^j$ along the trajectory refer to location of the centroid of the Voronoi cell corresponding to Agent 1 in the jth step. These are the points where search was performed by Agent 1. Although there are eight "deploy and search" steps in SDS, only five "o"s are visible along each agent's trajectory. This is because multiple searches have been performed at the second and third "o" since the centroids in successive steps were closer than a preset tolerance limit, and there was no movement of the agents in those deployment steps.

Fig. 1(c) shows the trajectory of Agent 2 moving toward the centroid corresponding to its Voronoi cell. Positions of Agent 2 are marked with "+", while "o" marks the centroids at successive time instances. Positions of Agent 2 in the first two time steps are marked as 1 and 2, while the centroids are marked as 1' and 2'. The agent tracks the changing centroid. Deployment stops and search is performed when the agent is sufficiently close to the corresponding centroid. One of the search instances is also marked, where, after search, in order to track the next centroid, Agent 2 takes an abrupt turn. This leads to non-smooth trajectories.

Fig. 1(d) illustrates the reduction in uncertainty density with the number of time steps. The uncertainty density remains constant during the process of deployment, between two consecutive searches. Whenever search is performed, the uncertainty density is reduced.

It is interesting that although the Voronoi partition-based strategy proposed in this paper is computationally complex, it results in collision free trajectories in a natural way, whereas non-Voronoi-based search strategies need additional collision avoidance schemes and require additional computations.

IV. Conclusion

The problem of multiagent search in an unknown environment is addressed in this paper. The lack of information in the search space was modeled as an uncertainty density distribution and a multiagent search strategy, namely, *sequential deploy and search* was proposed. It was shown that the centroidal Voronoi configuration with respect to the density as perceived by the sensors is a locally optimal configuration of the agents maximizing single step search effectiveness. A control law was proposed, which makes the agents move toward the respective centroids and the corresponding agent trajectories were shown to globally asymptotically converge to the centroidal Voronoi deployment configuration. The simulation results show the performance of the SDS strategy.

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References

[1] S. J. Benkoski, M. G. Monticino, and J. R. Weisinger, "A survey of the search theory literature," *Naval Research Logistics*, vol. 38, no. 4, pp. 469–494, 1991.
[2] D. Enns, D. Bugajski, and S. Pratt, "Guidance and control for cooperative search," in *Proc. Amer. Control Conf.*, Anchorage, AK, May 2002, pp. 1923–1029.
[3] S. V. Spires and S. Y. Goldsmith, "Exhaustive search with mobile robots along space filling curves," in *Collective Robotics, Lecture Notes in Artificial Intelligence, 1456*, A. Dragoul, M. Tambe, and T. Fukuda, Eds. Berlin, Germany: Springer-Verlag, 1998, pp. 1–12.
[4] P. Vincent and I. Rubin, "A framework and analysis to cooperative search using UAV swarms," in *Proc. ACM Symp. Appl. Comput.*, Nicosia, Cyprus, 2004, pp. 79–86.
[5] R. W. Beard and T. W. McLain, "Multiple UAV cooperative search under collision avoidance and limited range communication constraints," in *Proc. IEEE Conf. Decison Control*, Hawaii, Dec. 2003, pp. 25–30.
[6] M. Flint, E. F. A. Gaucherand, and M. Polycarpou, "Cooperative control for UAVs searching risky environments for targets," in *Proc. IEEE Conf. Decision Control*, Maui, HI, Dec. 2003, pp. 3568–3572.
[7] M. Polycarpou, Y. Yang, and K. Passino, "A cooperative search framework for distributed agents," in *Proc. IEEE Int. Symp. Intell. Control*, Mexico, Sep. 2001, pp. 1–6.
[8] Y. Yang, A. A. Minai, and M. M. Polycarpou, "Analysis of opportunistic method for cooperative search by mobile agents," in *Proc. IEEE Conf. Decision Control*, Las Vegas, NV, Dec. 2002, pp. 576–577.
[9] D. G. Rajnarayan and D. Ghose, "Multiple agent team theoretic decision making for searching unknown environment," in *Proc. IEEE Conf. Decision Control*, Maui, HI, Dec. 2003, pp. 2543–2548.
[10] R. F. Dell, J. N. Eagel, G. H. A. Martins, and A. G. Santos, "Using multiple searchers in constrained-path, moving-target search problems," *Naval Research Logistics*, vol. 43, pp. 463–480, 1996.
[11] P. B. Sujit and D. Ghose, "Negotiation schemes for multi-agent cooperative search," *Proc. IMech, Part G: J. Aerosp. Eng.*, vol. 223, no. 6, pp. 791–813, 2009.
[12] J. Cortés, S. Martínez, T. Karata, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Trans. Robot. Autom.*, vol. 20, pp. 243–255, 2004.
[13] K. R. Guruprasad and D. Ghose, "Deploy and search strategy for multi-agent systems using Voronoi partitions," in *Proc. 4th Int. Symp. Voronoi Diagrams in Sci. Eng. (ISVD 2007)*, Wales, Jul. 2007, pp. 91–100.
[14] P. K. Kundu and I. M. Cohen, *Fluid Mechanics*. Reading, MA: Academic Press, 2002.
[15] H. J. Marquez, *Nonlinear Control Systems—Analysis and Design*. New York: Wiley, 2003.
[16] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, pp. 1747–1767, 1999.
[17] J. Cortés, S. Martínez, and F. Bullo, "Spatially-distributed coverage optimization and control with limited-range interactions," *ESAIM: Control, Optimization and Calculus of Variations*, vol. 11, no. 4, pp. 691–719, 2005.
[18] E. B. Kosmatopoulos and M. A. Christodoulou, "Convergence properties of a class of learning vector quantization algorithms," *IEEE Trans. Image Process.*, vol. 5, pp. 361–368, Feb. 1996.
[19] J. C. Fort and G. Pages, "On the A.S. convergence of the Kohonen algorithm with a general neighborhood function," *Ann. Appl. Probability*, vol. 5, no. 4, pp. 1177–1216, 1995.