Concerning Channel Base Current Functions for Lightning Studies

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***Abstract*—Analysis of channel base current functions with specific reference to new channel base current (NCBC) function is reported. The main concern has been of adjusting the maximum time rate of change of lightning current $(di/dt)_{\max}$ along with time to crest t_m. A new ratio (maximum time rate of change of lightning current $(di/dt)_{\max}$ by average time rate of change of lightning current $(di/dt)_{\rm avg}$) has been defined. The critical numerical value for this ratio is identified as 1.7 for NCBC function. The performance of NCBC function is compared with Heidler's function in the wave front region.**

***Index Terms*—Base current, Heidler's function, lightning, maximum time rate of change of current, peak current, time to crest.**

I. Introduction

LIGHTNING electromagnetic field (LEMF) related research pertaining to modeling, begins with lightning channel base current as its first step (for engineering model) [1]. This involves identification and use of an appropriate mathematical function to represent the lightning channel base current. This function should account for realistic spatial and temporal variations associated with current development in the lightning channel. A detailed review of lightning return stroke models, highlighting the importance of engineering model, is found in [1] and [2]. Accounting for the variation of current along the channel and calculating the LEMF forms the second step of modeling.

A lightning flash will have one or more lightning strokes. Based on their characteristics the lightning return strokes are broadly grouped into first and subsequent return strokes. Practically measured data (field data) [3]–[7] related to lightning strokes have revealed that wave shape of a return stroke current can be viewed into five of its major constituents. They are

1) peak value of the current I_m;
2) maximum time rate of change of current $(di/dt)_{\max}$
3) time to crest t_m;
4) time to half of the peak value t_{50};
5) total charge Q.

Of the five parameters mentioned earlier I_m, t_m, and $(di/dt)_{\max}$ predominantly defines the wave front region. Many analytical functions are proposed in the literature to represent lightning channel base current. Prominent amongst them are

1) double exponential function (DEXP) [8];
2) Heidler's function [9];
3) new channel base current function (NCBC) [10].

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The DEXP function given by Bruce and Golde [8] is simple but gives an unrealistic convex wave front, resulting in the high value of current steepness at time $t = 0^+$ $\left((di/dt)\big|_{t=0^+} \neq 0\right)$. Heidler's function [9] could overcome the problem associated with DEXP, as it can reproduce more realistic concave to convex wave front change over, with $(di/dt)\big|_{t=0^+} = 0$. Heidler's function allows nearly independent adjustment of $I_m, (di/dt)_{\max}$, and charge Q. But when time to crest t_m also needs to be adjusted (independently), then the interdependence between I_m, t_m, and $(di/dt)_{\max}$ exists. Also, it is not time integrable [10]. In order to aid and simplify the tuning, Heidler *et al.* [9] have suggested graphical means of obtaining the function parameters. Recently, Javor and Rancic [10] have developed a NCBC function. The newly developed NCBC function is a time normalized function. Hence, the adjustment of peak current and time to crest is straight forward. It will give the realistic concave to convex wave-front change over. The NCBC function in addition provides the provision for adjusting the $(di/dt)_{\max}$ independent of t_m, as per ones requirement. The application of the NCBC function in representing the channel base currents for an IEC standard [6] current wave is clearly demonstrated in [10]. Because of NCBC function's flexibility in parameter adjustment for desired wave-shape, it is likely to gain acceptance.

This study tries to bring out the issues in relating I_m, t_m, and $(di/dt)_{\max}$ of the lightning channel base current by critically analyzing NCBC function. Also, the performance of NCBC function is compared with Heidler's function in the front region of the current wave.

II. Maximum and Average Current Steepness

The NCBC function has feature to adjust the $(di/dt)_{\max}$ independent of I_m and t_m. The effort here has been to have a relook at NCBC function in relating average and maximum time rate of currents. The NCBC function [10] is given by

$$i(0,t) = \begin{cases} I_m \tau^a \exp[a(1-\tau)] & 0 \le \tau \le 1 \\ I_m \sum_{i=1}^{n} c_i \tau^{b_i} \exp[b_i(1-\tau)] & 1 \le \tau \le \infty \end{cases} \quad (1)$$

where a, b_i are the function parameters, c_i are the weighing coefficients such that $\sum_{i=1}^{n} c_i = 1, \tau = t/t_m$ is the normalized time variable, t_m is the time taken by the current to reach its peak value I_m. The number of expressions chosen in the decaying part is specified by n.

As the $(di/dt)_{\max}$ occurs in the wave front portion, the analysis of the NCBC function is restricted to the region $0 \le \tau \le 1$. Using (1), it is possible to deduce an expression for $(di/dt)_{\max}$ [10] given as follows:

$$\left.\frac{di}{dt}\right|_{\max} = \frac{I_m}{t_m}\sqrt{a}(1 - 1/\sqrt{a})^{a-1}\exp\sqrt{a} \quad (2)$$

where I_m/t_m is the $(di/dt)_{\rm avg}$ (average rate of change of current). The plots of $(di/dt)_{\max}$ as a function of parameter "a"

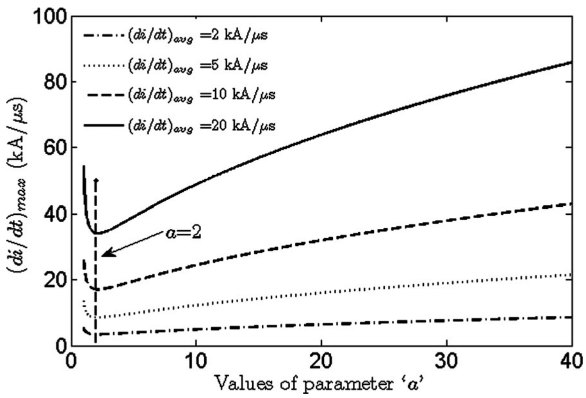

Fig. 1. Variation of $(di/dt)_{\max}$ with respect to parameter "a" for different values of ratio I_m/t_m.

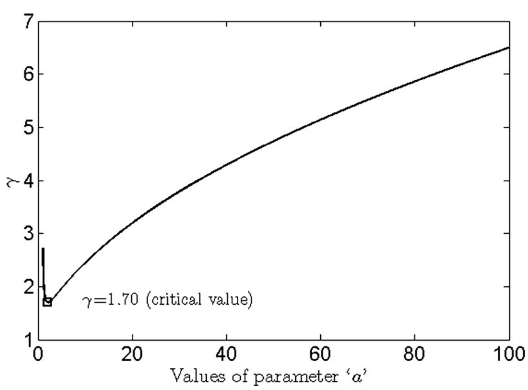

Fig. 2. Plot of "γ" as function of parameter "a."

with $(di/di)_{\rm avg}$ values as parameter are given in Fig. 1. In general, it can be observed that while using the NCBC function for every $(di/dt)_{\rm avg}$ there exists a critical $(di/dt)_{\max}$. Introducing a new ratio given by (3), "γ" will be purely the function of parameter "a"

$$\gamma = \frac{(di/dt)_{\max}}{(di/dt)_{\rm avg}} \tag{3}$$

The plot of "γ" as a function of "a" is as given in Fig. 2. Careful observation of Fig. 2 reveals that there exists a minimum "γ" corresponding to critical "a" of 2. The corresponding critical "γ" is given by

$$(\text{critical } \gamma)\ \gamma\big|_{a=2} = 1.70. \tag{4}$$

This imposes a restriction on the applicability of the NCBC function for any chosen $(di/dt)_{\max}$. The parameter "γ" while using the NCBC function cannot be less than 1.7. Meaningful values of parameter "a" of the NCBC function (mathematically) are possible with "γ" greater than or equal to 1.7 only. That is to say chosen $(di/dt)_{\max}$ is related to I_m and t_m through $(di/dt)_{\rm avg}$. The NCBC function provides a near independent adjustment of I_m, t_m, and $(di/dt)_{\max}$ by suitably varying the function parameter "a" (above a critical "γ" of 1.7). Independent adjustment of t_m in the NCBC function is an added advantage over Heidler's function, which also allows near independent adjustment of the I_m, $(di/dt)_{\max}$ and charge(Q) (except t_m).

TABLE I
NCBC Function Parameters to Reproduce the Channel Base Current Wave Shape

$I_m(kA)$	$t_m(\mu s)$	a	b_1	b_2	c_1	n
12	0.84	14.5	0.35	0.003	0.405	2

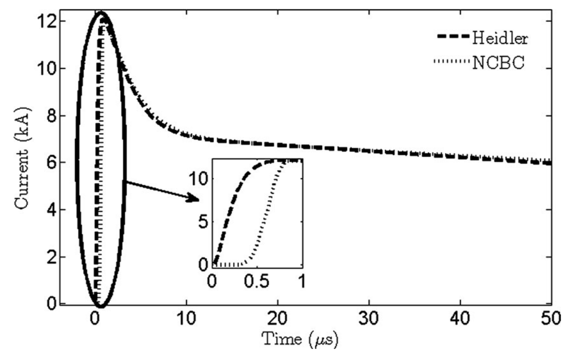

Fig. 3. Channel base current: comparison of Heidler's with the NCBC function (for $I_m = 12kA$, $(di/dt)_{\max}$=40 kA/μs).

The comparison of NCBC with Heidler's function is reported in the next section.

III. Comparison Between Heidler's and NCBC Functions

For the purpose of illustration, the typical lightning subsequent return stroke which is widely used in the literature [11], [12], (of peak current value 12 kA, time to crest 0.84 μs, and the maximum steepness of 40 kA/μs,) is considered. The NCBC function parameters for this desired current wave is adjusted and are given in Table I.

The aforementioned typical return stroke current wave is reproduced using Heidler's and the NCBC function (see Fig. 3). It clearly indicates that there is a significant a-sort-of delay in the front region for the NCBC function which is not seen in Heidler's function. Further this sort-of-delay depends on the maximum current steepness and the hence the value of parameter "a." This characteristic is expected as NCBC function tries to adjust $(di/dt)_{\max}$ by fixing I_m and t_m independently, by varying its function parameter "a." Fig. 4 shows such a comparison for a fixed $(di/dt)_{\rm avg}$ of 14.28 kA/μs for different values of $(di/dt)_{\max}$. For this comparison, the values $(di/dt)_{\max}$ chosen are corresponding to

1) limiting $(di/dt)_{\max}$ of NCBC function;
2) typical $(di/dt)_{\max}$ [6];
3) worst case $(di/dt)_{\max}$ [6].

With the increase in the $(di/dt)_{\max}$ a-sort-of inherent delay is introduced in the wave front region by the NCBC function. This can be seen from Fig. 4. Further this sort-of delay characteristic will replicate in the computed LEMF from NCBC function-based lightning current. Heidler's function-based computed LEMF at the ground level at 2 km away from the lightning channel by assuming perfect ground condition is compared with that of NCBC function-based LEMF in Figs. 5 and 6. The engineering model, modified transmission line with exponential (MTLE) decay is adopted for the computation of

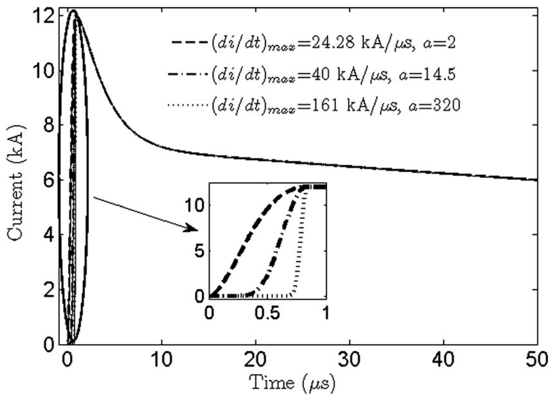

Fig. 4. NCBC function with different current steepness (for fixed $I_m = 12$ kA and $t_m = 0.84$ μs).

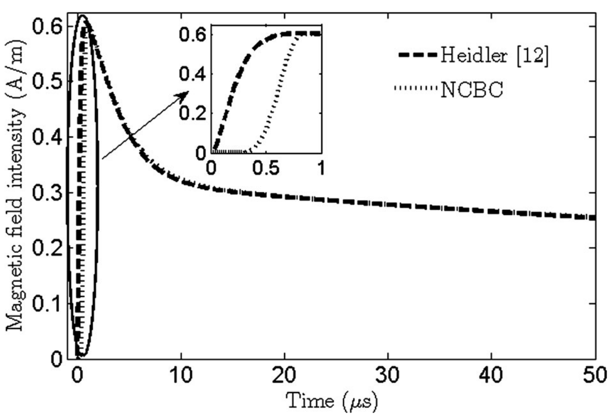

Fig. 5. Azimuthal magnetic field at 2 km from the lightning channel with the return subsequent stroke initiated at ground level.

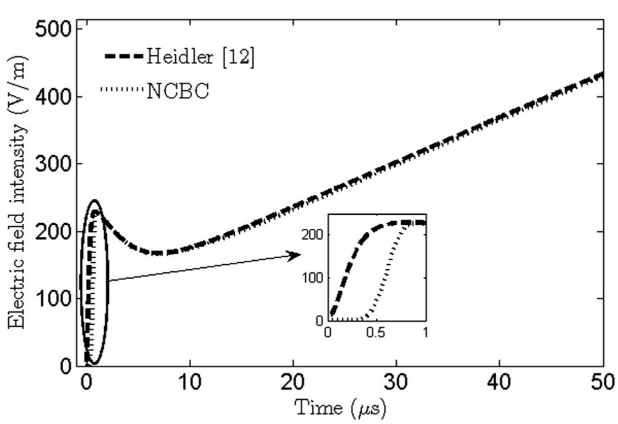

Fig. 6. Vertical electric field at 2 km from the lightning channel with the return subsequent stroke initiated at ground level.

LEMF. The method being one of the widely used methods for LEMF computations [11]–[15] is adopted in this study. The computed LEMFs front region is expanded and shown in the insets of Figs. 5 and 6. The insets clearly indicate a sort of delay introduced by the NCBC function. This delay is significant as it is comparable with the time to crest, at higher values of "γ." Heidler's function-based computation results shown in these figures are identical to those given in [12] (used as bench mark) and are reproduced for the sake of comparison.

IV. Summary

Function parameters adjustment for the given desired lightning current wave-shape using NCBC is straight forward. NCBC function can be used for independently adjusting $(di/dt)_{\max}, I_m$ and t_m of the desired current waveshape for any ratio "γ" greater than 1.7. Also, while adjusting the $(di/dt)_{\max}, I_m$, and t_m independently, the NCBC function shows a-sort-of delay characteristics in the front region of the current wave, which is significant as it is comparable with the time to crest t_m, for higher values of "γ." Thus, the ratio "γ" stresses the importance of relating $(di/dt)_{\max}, I_m$, and t_m.

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